A LOCAL-ETHER WAVE EQUATION AND THE CONSEQUENT ELECTROMAGNETIC FORCE LAW

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Abstract—The local-ether wave equation incorporating a nature frequency and the electric scalar potential is presented, from which the electrostatic force in conjunction with the inertial mass is derived. It is found that the inertial mass of a charged particle originates from the temporal variation of the associated matter wave. Further, the wave equation is extended by connecting the scalar potential to the augmentation operator which in turn is associated with the momentum operator and the velocity of source particles. From this local-ether wave equation, a first-order time evolution equation is derived, which in turn leads to the electromagnetic force law based on the augmented potentials. Under the low-speed condition, this law reduces to the modified Lorentz force law.

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1. INTRODUCTION

It is generally expected from intuition that the electromagnetic force exerted on a charged particle should remain unchanged as observed in different frames in uniform translational motion. However, it is well known that the famous Lorentz force law $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ for a particle of charge $q$ and velocity $\mathbf{v}$ is not invariant under Galilean transformations. To make the electromagnetic force be invariant, the Lorentz transformation of space and time seems to be the only possible approach. Recently, it is pointed out that the current associated with the magnetic force in the magnetic deflection or in motors is electrically neutralized. That is, the mobile charged particles which form the current are actually drifting in a matrix and the ions which constitute the matrix tend to electrically neutralize the mobile particles, such as electrons in a metal wire. Consequently, the velocity which determines the current density is the drift velocity of the mobile particles with respect to the neutralizing matrix. Thus a simple fact is unveiled: the neutralized current density is unchanged in different frames. In this view the effort to achieve the invariance of force law could be unnecessary. Further, based on the augmented potentials which are derived from the electric scalar potential by incorporating a velocity difference between involved particles, an electromagnetic force law has been proposed, which complies with Galilean transformations while it reduces to a familiar form under some common condition [1]. Thereby, we have presented a reinterpretation of this century-old issue about the invariance of electromagnetic force law from an entirely different approach.

In this new classical theory all of the involved position vectors, time derivatives, and velocities are referred specifically to their respective reference frames. Owing to this simple feature, the resulting electromagnetic force exerted on a charged particle remains unchanged when observed in different frames, as expected intuitively. Under the common low-speed condition where the source particles drift very slowly with respect to a matrix, this force law reduces to a form similar to the Lorentz force law. However, the fundamental modification is that the current density generating the magnetic vector potential, the time derivative applied to this potential in defining electric field, and the particle velocity connecting to the curl of this potential in the magnetic force are all referred specifically to the matrix frame. For quasi-static case where time derivative and propagation delay can be neglected, the electromagnetic force law becomes Galilean invariant.

The propagation of the potentials is supposed to follow the local-ether model recently presented in [2]. According to this model,
electromagnetic wave can be viewed as to propagate via a medium like the ether. However, the ether is not universal. It is supposed that in the region under sufficient influence of the gravity due to the Earth, the Sun, or another celestial body, there forms a local ether which in turn moves with the gravitational potential of the respective body. For earthbound wave, the medium is the earth local ether which is stationary in an ECI (earth-centered inertial) frame, while the sun local ether for interplanetary wave is stationary in a heliocentric inertial frame. Consequently, earthbound experiments can depend on earth’s rotation but are entirely independent of earth’s orbital motion around the Sun or whatever. This local-ether model has been adopted to account for the effects of earth’s motions in a wide variety of propagation phenomena, particularly the Sagnac pseudorange correction in GPS (global positioning system), the time comparison by intercontinental microwave link, and the echo time in interplanetary radar [2].

Further, matter wave is supposed to follow the local-ether propagation model and is governed by a wave equation incorporating a natural frequency and the gravitational and the electrical scalar potentials [3, 4]. Accordingly, the position vectors, time derivatives, and velocities in this wave equation are all referred specifically to an ECI frame for earthbound phenomena. Under the ordinary condition of low particle speed, the local-ether wave equation has been shown to lead to a unified quantum theory of gravitational and electromagnetic forces [3]. Furthermore, it leads to the important consequence that the gravitational mass associated with the gravitational force is identical to the inertial mass under the influence of the electrostatic force. In this investigation, the local-ether wave equation is extended in such a way that the electric scalar potential is made to connect to the augmentation operator which in turn is associated with a velocity difference between involved particles. This augmentation operator will be proposed in such a way that the wave equation will preserve the local-ether feature. Then we explore the relations of the local-ether wave equation with the electromagnetic force law and the augmented potentials.

2. LOCAL-ETHER WAVE EQUATION AND EVOLUTION EQUATION

It is supposed that a charged particle contributes to the electric scalar potential $\Phi$ which in turn will be experienced by and then will affect another charged particle under consideration called the effector. As well as electromagnetic wave, matter wave associated with a particle
is supposed to follow the local-ether propagation model. Accordingly, it is postulated that under the influence of the electric scalar potential \( \Phi \), the matter wave \( \Psi \) of the effector particle is governed by the nonhomogeneous wave equation proposed to be

\[
\left\{ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} \Psi(r, t) = \frac{\omega_0^2}{c^2} \left\{ 1 + \frac{2}{\hbar \omega_0} q \Phi(r, t) \right\} \Psi(r, t),
\]

where \( c \) is the speed of light, the natural frequency \( \omega_0 \) as well as the charge \( q \) is an inherent constant of the effector particle, and the position vector \( r \) and the time derivative are referred to the associated local-ether frame.

The incorporation of the factor \( \hbar/2 \), Planck's constant divided by \( 4\pi \), in the potential term may be not so obvious at the first glance. However, this factor is indispensable if the proposed wave equation is expected to lead to the well-studied electrostatic force in terms of potential \( \Phi \). Anyway, it makes the potential term \( 2q\Phi/\hbar \omega_0 \) dimensionless. The physical meaning and the actual value of the natural frequency of a charged particle are yet to be explored from the consequences derived from the wave equation. If the natural frequency \( \omega_0 \) is zero, this equation reduces to a homogeneous wave equation which in turn implies that wave propagates at the speed \( c \) with respect to the local-ether frame. Obviously, this wave equation provides the basis of the local-ether propagation model of electromagnetic wave discussed in [2].

Quantitatively, the electric scalar potential \( \Phi \) due to a single source particle of charge \( q \) and located at \( r' \) at instant \( t' \) is given by

\[
\Phi(r, t) = \frac{1}{4\pi \varepsilon_0} \frac{q}{R},
\]

where \( t = t' + R/c \), \( R/c \) is the propagation delay time from the source point \( r' \) at instant \( t' \) to the field point \( r \) at instant \( t \), and the propagation range \( R = |r - r'| \). The potential due to an ensemble of source particles can be given by superposition. Based on the local-ether model, the position vectors \( r' \) and \( r \) and hence the propagation speed \( c \) are also referred specifically to the local-ether frame. Thus the potential propagates at the speed \( c \) with respect to the local-ether frame away from the wave emission position, independent of the motions of source and effector. It is noted that in the potential \( \Phi \) and the wave equation, the involved position vectors, time derivatives, and velocities are all referred to the local-ether frame. For static or quasi-static case where the propagation delay can be neglected, the propagation range \( R \) reduces to the separation distance between the source and the field points at a given instant \( t \).
Consider the ordinary case where the scalar potential \( \Phi \) and the spatial rate of variation of \( \Psi \) are weak. Thus the temporal variation of \( \Psi \) is close to that of the harmonic \( e^{-i\omega_0 t} \) and then the wavefunction can be given as \( \Psi(\mathbf{r}, t) = \psi(\mathbf{r}, t)e^{-i\omega_0 t} \), where \( \psi \) is a weak function of time. Then its second time derivative becomes

\[
\frac{\partial^2}{\partial t^2} \Psi(\mathbf{r}, t) = \left\{ \frac{\partial^2}{\partial t^2} \psi(\mathbf{r}, t) - i2\omega_0 \frac{\partial}{\partial t} \psi(\mathbf{r}, t) - \omega_0^2 \psi(\mathbf{r}, t) \right\} e^{-i\omega_0 t}. \tag{3}
\]

As the temporal variation of \( \psi \) is relatively weak, the second derivative \( \partial^2 \psi / \partial t^2 \) can be neglected. Thereby, the proposed wave equation can be approximated to the first-order time evolution equation in terms of the reduced wavefunction \( \psi \) as

\[
\frac{\partial}{\partial t} \psi(\mathbf{r}, t) = i \frac{c^2}{2\omega_0} \nabla^2 \psi(\mathbf{r}, t) - i \frac{1}{\hbar} q\Phi \psi(\mathbf{r}, t), \tag{4}
\]

where the two major terms \( \omega_0^2 \psi \) cancel out and the time derivative is referred to the local-ether frame. This evolution equation states that the temporal variation of wavefunction \( \psi \) is determined by its spatial variation and the potential. Given the wavefunction \( \psi \) at a particular instant, the space-time behavior of the wavefunction after that instant is then governed by the evolution equation. If the potential is static, the space-time wavefunction is then determined from its starting spatial distribution.

3. DERIVATION OF ELECTROSTATIC FORCE AND MASS

As in quantum mechanics, it is supposed that the velocity \( \mathbf{v}_e \) of the effector particle is given by the time derivative of expectation value of its position vector \( \mathbf{r} \). That is,

\[
\mathbf{v}_e = \frac{d \langle \mathbf{r} \rangle}{dt}, \tag{5}
\]

where the position vector \( \mathbf{r} \) and hence the velocity \( \mathbf{v}_e \) are referred to the local-ether frame and the expectation value \( \langle O \rangle \) of an operator \( O \) is evaluated in terms of the reduced wavefunction \( \psi \) as \( \langle O \rangle = \int \psi^* O \psi d\mathbf{r} \). It is noted that as velocity \( \mathbf{v}_e \) represents Newtonian relative velocity between the effector and the local-ether frame, it is Galilean invariant.

It can be shown that for two arbitrary wavefunctions \( \psi_a \) and \( \psi_b \),

\[
\int (\tilde{H} \psi_a)^* \psi_b d\mathbf{r} = -\int \psi_a^* \tilde{H} \psi_b d\mathbf{r}, \tag{6}
\]
where $\tilde{H}$ represents the sum of all the operators on the right-hand side of the evolution equation (4), that is, $\tilde{H}\psi = \partial\psi/\partial t$. Thus, by a direct expansion, the time derivative of the expectation value of a time-independent operator $O$ can be given by

$$\frac{d\langle O \rangle}{dt} = \langle [O, \tilde{H}] \rangle,$$

(7)

where the commutator $[O, \tilde{H}] = O\tilde{H} - \tilde{H}O$ and the time derivative is referred to the associated local-ether frame, identical to the one of the time derivative in the evolution equation. It is straightforward to show the two useful commutator relations

$$[r, f(\nabla - s)^2] = f[r, (\nabla - s)^2] = -2f(\nabla - s)$$

(8)

and

$$[\nabla, f(\nabla - s)^2] = (\nabla f)(\nabla - s)^2,$$

(9)

where $f$ is an arbitrary scalar function of space and $s$ is an arbitrary constant vector.

Based on the evolution equation (4), a use of the commutator relation (8) leads to that the effector velocity is given by

$$v_e = \langle [r, \tilde{H}] \rangle = -i\frac{c^2}{\omega_0} \langle \nabla \rangle.$$

(10)

It is seen that the effector velocity is proportional to the expectation value of the del operator and thus is associated with the spatial rate of variation of the wavefunction. Thereby, a zero or weak spatial rate of variation of the wavefunction corresponds to that the particle is stationary or moving at a low speed with respect to the local-ether frame, respectively. Thus the evolution equation (4) is valid only for a low-speed effector under a weak potential.

Further, the acceleration of the effector particle can be given in terms of the time derivative of this expectation value as

$$a = -i\frac{c^2}{\omega_0} \frac{d}{dt} \langle \nabla \rangle = -i\frac{c^2}{\omega_0} \langle [\nabla, \tilde{H}] \rangle.$$

(11)

It is straightforward to show that the acceleration is given by

$$a = -\frac{c^2 q}{\omega_0 \hbar} \nabla \Phi.$$

(12)

It is noted that the acceleration is proportional to the gradient of potential $\Phi$. Physically, this relation states that the spatial variation
of potential $\Phi$ tends to cause a change in the spatial variation of wavefunction $\psi$ and then a change in the effector velocity. It is also noted that the acceleration is inversely proportional to the natural frequency. Thus the natural frequency just plays the important role of inertial mass.

Based on the electrostatic force given by $\mathbf{F} = -q \nabla \Phi$ and on Newton’s second law of motion in classical mechanics, the acceleration of a charged particle under the influence of electric scalar potential is known as

$$a = -\frac{1}{m_0} q \nabla \Phi,$$  \hspace{5cm} (13)

where $m_0$ is the rest mass of accelerated particle. By comparing the preceding two formulas of acceleration, the local-ether wave equation can be in accord with classical mechanics and electrostatics, if the rest mass $m_0$ and the natural frequency $\omega_0$ are related to each other in the familiar form of

$$m_0 = \frac{\hbar}{c^2} \omega_0.$$  \hspace{5cm} (14)

This frequency-mass relation links the inertial mass of a particle to the natural frequency of the associated matter wave and unveils the physical origin of inertial mass as a wave phenomenon. In other words, the **inertial mass of the effector particle originates from the temporal variation of the wavefunction $\Psi$** (rather than that of the reduced wavefunction $\psi$).

Precisely, the natural frequency $\omega_0$ multiplied by the constant ($\hbar/c^2$) is just the inertial mass $m_0$ of the particle under the influence of the electric scalar potential $\Phi$, where it is supposed that the potential is weak and the particle is stationary or moving slowly with respect to the local-ether frame. Planck’s constant $\hbar$ foresightedly incorporated in the potential term in the local-ether wave equation (1), when divided by $c^2$, serves to adapt the natural frequency $\omega_0$ to the rest mass $m_0$ which has been measured in kilogram or the like long before the introduction of the charge $q$ and the potential $\Phi$. It is known that the rest mass of an electron is given as $9.107 \times 10^{-31}$ kg, the permittivity in the electric scalar potential is defined as $\varepsilon_0 = 1/\mu_0 c^2$, and the permeability in turn is defined as $\mu_0 = 4\pi \times 10^{-7}$ (kg·m/C²). Thereby, according to the acceleration formula (13), the charge of an electron would be $1.602 \times 10^{-19}$ (C) by a measurement of acceleration.

In terms of the rest mass $m_0$, the first-order time evolution equation (4) can be rewritten as

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m_0} \nabla^2 \psi(\mathbf{r}, t) + q\Phi \psi(\mathbf{r}, t).$$  \hspace{5cm} (15)
This evolution equation looks like the famous Schrödinger's equation with interaction $q\Phi$, except the reference frame. The foresighted incorporation of Planck's constant $\hbar$ in the potential term makes the time derivative and the Laplacian operator in the evolution equation incorporate a factor of $\hbar$ and $\hbar^2$, respectively, just as in Schrödinger's equation.

4. LOCAL-ETHER WAVE EQUATION WITH AUGMENTATION OPERATOR

In order to derive the whole electromagnetic force, it is proposed that the wave equation is modified by connecting the potential $\Phi$ to a dimensionless operator $U$. For the electric scalar potential $\Phi$ due to source particles of a given velocity $v_s$ with respect to the local-ether frame, it is postulated that the local-ether wave equation incorporates the operator $U$. That is,

$$\left\{ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} \Psi(\mathbf{r}, t) = \frac{\omega_0^2}{c^2} \left\{ 1 + \frac{2}{\hbar \omega_0} q \Phi(\mathbf{r}, t)(1 + U) \right\} \Psi(\mathbf{r}, t), \quad (16)$$

where operator $U$ is derived from the Laplacian operator and is given by

$$U = \frac{1}{2c^2} \left( -i \frac{c^2}{\omega_0} \nabla - v_s \right)^2. \quad (17)$$

The operator $U$ tends to enhance the effect of the electric scalar potential and hence is named the augmentation operator. For source particles of various velocities, the whole potential term should be given by superposition. As the source velocity is referred specifically to the local-ether frame, the incorporation of the augmentation operator does not spoil the local-ether feature of the wave equation. This local-ether wave equation made its debut in [5].

In terms of the rest mass $m_0$, the augmentation operator $U$ becomes

$$U = \frac{1}{2c^2} \left( \frac{p}{m_0} - v_s \right)^2, \quad (18)$$

where the operator $p = -i\hbar \nabla$. From (10), it is seen that $\langle p \rangle = m_0 v_e$. As the operator $p$ is associated with the momentum of the effector, the augmentation operator $U$ is associated with a squared velocity difference between the effector and the source normalized with respect to $c$. The speeds of these particles are supposed to be much lower
than that of light, as they are ordinarily. Thus the modification with the augmentation operator is of the second order of normalized speed (with respect to \( c \)) and is actually quite slight in magnitude.

Again, suppose that both the electric scalar potential \( \Phi \) and the spatial variation of wavefunction \( \Psi \) are weak. Hence the effector speed \( v_e \) with respect to the local-ether frame is much lower than \( c \). Thus the temporal variation of \( \Psi \) is still close to that of the time harmonic \( e^{-i\omega_0 t} \) as \( \Psi(r, t) = \psi(r, t)e^{-i\omega_0 t} \), where the reduced wavefunction \( \psi \) has a weak temporal variation. By neglecting the second derivative \( \partial^2 \psi / \partial t^2 \), the local-ether wave equation can be approximated to the first-order time evolution equation in terms of the reduced wavefunction \( \psi \). That is,

\[
i2\omega_0 \frac{1}{c^2} \frac{\partial}{\partial t} \psi(r, t) = -\nabla^2 \psi(r, t) + \frac{2\omega_0}{\hbar c^2} q\Phi(1 + U)\psi(r, t),
\]

where the position vector \( r \) and the time derivative are referred to the local-ether frame. In terms of the rest mass \( m_0 \), the first-order time evolution equation for an effector in the presence of the potential \( \Phi \) due to source particles moving at a given velocity \( v_s \) becomes

\[
i\hbar \frac{\partial}{\partial t} \psi(r, t) = \frac{1}{2m_0} \mathbf{p}^2 \psi(r, t) + q\Phi(r, t) \left\{ 1 + \frac{(\mathbf{p} - m_0 v_s)^2}{2m_0^2 c^2} \right\} \psi(r, t).
\]

This evolution equation is somewhat similar to Schrödinger’s equation. However, the fundamental difference is that the position vectors, the time derivative, the propagation velocity of the potential, and the source velocity are all referred specifically to the local-ether frame. Furthermore, the electric scalar potential connects to the augmentation operator. This augmentation in potential will be shown to correspond to the magnetic vector potential.

5. DERIVATION OF LOCAL-ETHER ELECTROMAGNETIC FORCE

Based on the evolution equation just developed from the proposed local-ether wave equation, we go on to derive the consequent electromagnetic force law. Again, the effector velocity \( v_e \) is given by the time derivative of expectation value of the position operator \( r \) and hence by the commutator between \( r \) and Hamiltonian \( H \) as

\[
v_e = \frac{d}{dt} \langle r \rangle = \frac{1}{i\hbar} \langle [r, H] \rangle,
\]

where the position vector \( r \), the time derivative, and hence the effector velocity \( v_e \) are referred to the local-ether frame, the expectation
value is evaluated in terms of the reduced wavefunction $\psi$, and the Hamiltonian $H$ represents the sum of all the operators on the right-hand side of the evolution equation (20), that is, $H\psi = i\hbar \partial \psi / \partial t$.

A use of commutator relation (8) immediately leads to

$$v_e = \frac{1}{m_0} \left\{ \langle p \rangle + \frac{1}{m_0 c^2} \langle q \Phi (p - m_0 v_s) \rangle \right\}. \quad (22)$$

It is noted that by virtue of the momentum operator $p$ incorporated in the augmentation operator $U$, the scalar potential $\Phi$ tends to affect the effector velocity $v_e$. For a localized particle over which the spatial variation of potential $\Phi$ can be neglected, the effector velocity then becomes

$$v_e = \frac{1}{m_0} \left\{ \langle p \rangle + q \Phi_e \frac{1}{m_0 c^2} (\langle p \rangle - m_0 v_s) \right\}, \quad (23)$$

where $\Phi_e$ denotes the potential experienced by the effector and hence is equal to the functional value of the potential $\Phi$ at the instantaneous location of the effector particle.

According to the preceding velocity formula, define the augmented vector potential $\vec{A}$ originating from source particles moving at a velocity $v_s$ in terms of the corresponding potential $\Phi$ as

$$\vec{A}(r, t) \triangleq -\Phi(r, t) \frac{1}{m_0 c^2} (\langle p \rangle - m_0 v_s). \quad (24)$$

Then the effector velocity can be given by

$$v_e = \frac{1}{m_0} \left\{ \langle p \rangle - q \vec{A}_e \right\}, \quad (25)$$

where $\vec{A}_e$ denotes the augmented potential experienced by the effector and hence is equal to the functional value of the potential $\vec{A}$ at the instantaneous location of the effector. It is noted that the effector velocity is $\langle p \rangle / m_0$ modified by a quantity of $q \vec{A}_e / m_0$. It is seen that the effector velocity is associated with the augmented vector potential, which in turn is associated with the effector velocity itself. The fact that the expectation value $\langle p \rangle$ and the experienced potential $\vec{A}_e$ are independent of reference frame is consistent with the Galilean invariance of the effector velocity $v_e$.

Further, the acceleration $a$ of the effector or the second time derivative of expectation value of the position vector $r$ is then given by

$$a = \frac{1}{m_0} \left\{ \frac{d}{dt} \langle p \rangle - q \frac{d}{dt} \vec{A}_e \right\}, \quad (26)$$
where the time derivative $d\tilde{A}_e/dt$ represents the time rate of change in the augmented potential experienced by the effector. Note that $\tilde{A}_e$ is not a function of space and its value and time derivative are independent of reference frame. As the effector is moving at a velocity $\mathbf{v}_s$ with respect to the local-ether frame and is supposed to be located at $\mathbf{r}$ at instant $t$, $d\tilde{A}_e/dt$ is equal to $(\partial \tilde{A}(\mathbf{r}, t)/\partial t)_e$, the time rate of change of the augmented potential in the effector frame (with respect to which the effector is stationary) at the location of the effector. This derivative is due partly to the variations in velocities of source and effector and partly to the relative displacement between these particles which affects the actually experienced potential $\Phi_e$.

Furthermore, the time derivative of expectation value of the momentum operator $\mathbf{p}$ can be given by

$$
\frac{d}{dt} \langle \mathbf{p} \rangle = \frac{1}{i\hbar} \langle [\mathbf{p}, H] \rangle = -q(\nabla \Phi) \{1 + \langle U \rangle\}, \quad (27)
$$

where the commutator relation (9) has been made use of. According to the preceding formula, define the augmented scalar potential $\tilde{\Phi}$ originating from source particles moving at a velocity $\mathbf{v}_s$ in terms of the corresponding potential $\Phi$ as

$$
\tilde{\Phi}(\mathbf{r}, t) \triangleq \Phi(\mathbf{r}, t) \{1 + \langle U \rangle\}. \quad (28)
$$

It is seen that the augmented scalar potential is the electric scalar potential with a fractional increment given by the expectation value of the augmentation operator. Then the time derivative of expectation value of momentum operator $\mathbf{p}$ can be given in terms of the augmented scalar potential as

$$
\frac{d}{dt} \langle \mathbf{p} \rangle = -q \nabla \tilde{\Phi}. \quad (29)
$$

This relation shows that the effect of the spatial variation of potential $\tilde{\Phi}$ on the change in the spatial variation of wavefunction $\psi$ is enhanced slightly by the augmentation operator.

Thereby, the acceleration of the effector can be given in terms of the augmented scalar potential $\tilde{\Phi}$ and the augmented vector potential $\tilde{\mathbf{A}}$. Based on the classical Newton's second law of motion with the inertial mass of the effector being the rest mass $m_0$, the electromagnetic force exerted on the effector can be given in terms of the augmented potentials. That is,

$$
\mathbf{F}(\mathbf{r}, t) = q \left\{ -\nabla \tilde{\Phi}(\mathbf{r}, t) - \left( \frac{\partial}{\partial t} \tilde{\mathbf{A}}(\mathbf{r}, t) \right)_e \right\}. \quad (30)
$$
In this force law all of the involved position vectors, time derivatives, and velocities are referred specifically to their respective reference frames. Consequently, the force exerted on an effector remains unchanged when observed in different frames, as expected intuitively. It is noted that the augmented potentials incorporate the expectation value of the momentum operator and hence tend to be different for different effectors. Moreover, it is noted that both the augmented potentials originate from the electric scalar potential and hence propagate together at exactly the same velocity.

6. AUGMENTED POTENTIALS UNDER WEAK POTENTIAL AND NEUTRALIZATION

Remark that the time evolution equation and hence its consequences of effector velocity, acceleration, and force are derived under the weak-potential condition. Under this condition, the modification term \( q\vec{A}_e/m_0 \) in the effector velocity \( \vec{v}_e \) is weaker than the major term by a factor of about \( |q\Phi|/m_0c^2 \), which is then much less than unity. Thus, under the ordinary conditions of weak potential and low particle speeds, the effector velocity with respect to the local-ether frame can be approximated to \( \vec{v}_e = \langle \vec{p} \rangle /m_0 \). Thereby, the augmented scalar potential \( \tilde{\Phi} \) (28) and the augmented vector potential \( \tilde{\vec{A}} \) (24) due to source particles moving at a given velocity \( \vec{v}_s \) are related to the electric scalar potential \( \Phi \) as

\[
\tilde{\Phi}(\vec{r}, t) = \left( 1 + \frac{v_{es}^2}{2c^2} \right) \Phi(\vec{r}, t) \tag{31}
\]

and

\[
\tilde{\vec{A}}(\vec{r}, t) = -\frac{v_{es}}{c^2} \tilde{\Phi}(\vec{r}, t), \tag{32}
\]

where \( v_{es} = |\vec{v}_{es}| \) and the velocity difference \( \vec{v}_{es} = \vec{v}_e - \vec{v}_s \).

The augmented potentials originating from an ensemble of mobile source particles of various velocities, together with the corresponding electromagnetic force, can then be given simply by superposition. Due to the sources of charge density \( \rho_v \), the augmented potentials experienced by an effector located at position \( \vec{r} \) at instant \( t \) are then given by superposition as

\[
\tilde{\Phi}(\vec{r}, t) = \frac{1}{4\pi\varepsilon_0} \int \left( 1 + \frac{v_{es}^2}{2c^2} \right) \frac{\rho_v(\vec{r}', t - R/c)}{R} dv'
\tag{33}
\]
and

$$\vec{A}(\vec{r}, t) = \frac{-1}{4\pi\epsilon_0 c^2} \int \frac{\vec{v}_{es}\rho_v(\vec{r}', t - R/c)}{R} dv',$$

(34)

where $\vec{v}_e$ is the effector velocity at instant $t$, $\vec{v}_s$ is the various velocity of the source particles located at $\vec{r}'$ at an earlier instant $t' = t - R/c$, and the propagation range $R (= |\vec{r} - \vec{r}'|)$ is the distance from the source point $\vec{r}'$ at instant $t'$ to the field point $\vec{r}$ at instant $t$ with respect to the local-ether frame. It is seen that both the augmented potentials depend on the velocity difference $\vec{v}_{es}$ and the change density $\rho_v$. For quasi-static case where the propagation delay time $R/c$ can be neglected or for the case where velocities $\vec{v}_e$ and $\vec{v}_s$ are fixed, the velocity difference $\vec{v}_{es}$ then becomes the Newtonian relative velocity between the effector and the source. The derived electromagnetic force law (30) together with the augmented potentials (33) and (34) has been treated in [1] as fundamental postulates to derive the modified Lorentz force law. It is seen that potential $\vec{A}$ is associated with minus the derivative of potential $\Phi$ with respect to the effector speed $v_e$.

The augmented potentials given in (33) and (34) can be further simplified. As discussed in [1], consider the ordinary case where the ensemble of source particles are moving in a matrix which tends to neutralize them. Suppose the neutralizing matrix is of an arbitrary charge density $\rho_m$ and moves as a whole at a velocity $\vec{v}_m$ with respect to the local-ether frame. Under the ordinary low-speed condition where all the involved particles move slowly with respect to the local-ether frame and the sources drift very slowly with respect to the matrix frame ($v_{sm} \ll c$ and $v_{em}, v_m \ll c$), it can be shown that the augmented potentials can be simplified to [1]

$$\Phi(\vec{r}, t) = \Phi(\vec{r}, t) - \vec{v}_{em} \cdot \vec{A}(\vec{r}, t)$$

(35)

and

$$\vec{A}(\vec{r}, t) = \vec{A}(\vec{r}, t),$$

(36)

where the velocity difference $\vec{v}_{em} = \vec{v}_e - \vec{v}_m$. The electric scalar potential $\Phi$ and the magnetic vector potential $\vec{A}$ in turn are given by

$$\Phi(\vec{r}, t) = \frac{1}{\epsilon_0} \int \frac{\rho_n(\vec{r}', t - R/c)}{4\pi R} dv'$$

(37)

and

$$\vec{A}(\vec{r}, t) = \frac{1}{\epsilon_0 c^2} \int \frac{\vec{J}_n(\vec{r}', t - R/c)}{4\pi R} dv',$$

(38)
where potential $\Phi$ is due to the net charge density $\rho_n = \rho_v + \rho_m$, potential $A$ is due to the neutralized current density $J_n = v_{sm}\rho_v$, and $v_{sm} (= v_s - v_m)$ is the Newtonian relative velocity of the source particle with respect to the matrix. Thereby, under the ordinary weak-potential and low-speed conditions, the electromagnetic force law (30) for a particle of charge $q$ and inertial mass $m_0$ can be given in terms of the local-ether potentials $\Phi$ and $A$. That is,

$$F(r, t) = q \left\{ -\nabla \Phi(r, t) - \left( \frac{\partial}{\partial t} A(r, t) \right)_m + v_{em} \times \nabla \times A(r, t) \right\}, \quad (39)$$

where we have made use of the Galilean transformation: $(\partial A/\partial t)_e = (\partial A/\partial t)_m + (v_{em} \cdot \nabla) A$ and $(\partial/\partial t)_e$ and $(\partial/\partial t)_m$ denote the time derivatives with respect to the effector and the matrix frames and are taken under constant $(r - v_e t)$ and $(r - v_m t)$, respectively, as $r$ is referred to the local-ether frame.

As discussed in [1], the preceding formula presents modifications of the Lorentz force law. The fundamental modifications are that the current density generating potential $A$, the time derivative applied to potential $A$, and the effector velocity before $\nabla \times A$ are all referred specifically to the matrix frame and that the propagation speed of potentials $\Phi$ and $A$ is referred specifically to the local-ether frame. Moreover, as the effector and the matrix speeds are low, the variation in the Sagnac effect is neglected, where the Sagnac effect is associated with the shift in the propagation range and delay time due to the movement of effector with respect to the local-ether frame during wave propagation and is proportional to the normalized speed [2]. Thereby, the preceding equation is identical to the Lorentz force law, if the latter is observed in the matrix frame as done in common practice. In other words, the Lorentz force law has some hidden restrictions. That is, the reference frame is actually referred to the matrix frame, the effector and the matrix speeds are low, the drift speed is very low, and the corresponding variation in the Sagnac effect is omitted. However, these conditions are so common as to be ignored easily.

7. CONCLUSION

Based on the local-ether wave equation incorporating a nature frequency and the electric scalar potential, the electrostatic force in conjunction with the inertial mass of the effector particle is given from the acceleration derived from a quantum-mechanical approach. It is found that the inertial mass of a low-speed particle is just the natural frequency of the associated matter wave, aside from a scaling factor.
This frequency-mass relation unveils the physical origin of inertial mass as the temporal variation of wavefunction.

Further, the local-ether wave equation is extended by connecting the electric scalar potential to the augmentation operator which in turn is associated with the momentum operator and the source velocity. The position vectors, the time derivative, the source velocity, and the propagation velocity of potential are referred specifically to the local-ether frame. As a consequence, the derived electromagnetic force remains unchanged when observed in different frames, as expected intuitively. Under the ordinary conditions of weak potential and low particle speeds, the local-ether wave equation leads to a first-order time evolution equation for a harmonic-like wavefunction.

From the evolution equation, it is shown that the electromagnetic force exerted on an effector can be given in terms of the augmented potentials which in turn incorporate the velocity difference between effector and source particles. Both the augmented potentials originate from the electric scalar potential and hence propagate together at exactly the same velocity. Thereby, the local-ether wave equation provides a quantum foundation for the electromagnetic force law based on the augmented potentials, which in turn leads to the modifications of the Lorentz force law under the common low-speed condition.

REFERENCES


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