A Wave Interpretation of the Compton Effect
As a Further Demonstration of the Postulates of de Broglie

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Abstract – The Compton effect is commonly cited as a demonstration of the particle feature of light, while the wave nature of matter has been proposed by de Broglie and demonstrated by Davisson and Germer with the Bragg diffraction of electron beams. In this investigation, we present an entirely different interpretation of the Compton effect based on the postulates of de Broglie and on an interaction between electromagnetic and matter waves. The speeds of interacting electrons in the Compton scattering are quite fast and its mechanism relies heavily on the mass variation. Thus, based on this wave interpretation, the Compton effect can be viewed as a further demonstration of the postulates of de Broglie for high-speed particles. In addition to the scattered wave, a direct radiation depending on the mass variation is predicted, which provides a means to test the wave interpretation.

1. Introduction

In 1923 Compton found that the x rays scattered from free electrons shift in wavelength. In Compton's explanation of the scattering, the collision between a photon and an electron in conjunction with the conservation of energy and momentum is adopted [1]. Thus the Compton effect is commonly cited as a demonstration of the corpuscular picture of light [2, 3]. On the other hand, in 1924 de Broglie initiated the concept of wave nature of matter with the postulate that a particle is associated with a matter wave of which the frequency and the wavelength are related to the energy and the momentum of the particle, respectively [3]. This hypothesis of matter wave led to the introduction of Schrödinger's equation, the Klein-Gordan equation, and the Dirac equation which play the fundamental role in quantum mechanics. Shortly, in 1927, the matter wavelength was demonstrated by Davisson and Germer with the Bragg diffraction of electron beams from a crystal [3]. More recently, various experiments of quantum interference between matter waves of two coherent beams of electrons, neutrons, or atoms have been reported to demonstrate the Bragg reflection, the double-slit diffraction, the gravitational effect, and the Sagnac effect [4, 5]. Particularly, the effect of earth's rotation has been detected by the neutron interferometry where the Bragg reflection from slabs of silicon crystal is used to form a closed path for neutron beams to interfere [5].

In this investigation, based on the postulates of de Broglie, we present an entirely different interpretation of the Compton effect by dealing with an interaction between electromagnetic and matter waves. Moreover, it is shown that the postulates of de Broglie themselves can be derived from the dispersion of matter wave which in turn is governed by the Klein-Gordan equation. Under the influence of electromagnetic waves, electrons are accelerated and a mixed state of matter wave of the interacting electrons is formed during their state transition. This mixed state leads to a space- and time-varying medium, from which electromagnetic waves are scattered. Then the Compton effect corresponds to a constructive interference of electromagnetic waves which results in dominant scattered waves among various other scattered waves and direct radiation from the medium. Thereby, the

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Compton effect is envisaged as the constructive scattering of electromagnetic waves from a space- and time-varying medium due to the mixed state, without explicit resort to the conservation laws for energy and momentum. Similar scattering mechanisms can also be used to interpret the Bragg reflection and the Raman scattering from a crystal. The particle speeds in the Compton effect can be much closer to the speed of light than those in the aforementioned quantum-interference experiments and its mechanism relies heavily on the mass variation. Thus, based on this wave interpretation, the Compton effect can be viewed as a further demonstration of the postulates of de Broglie for high-speed particles.

2. Compton Shift

When a beam of x rays of a certain wavelength is incident upon a target made of graphite, the scattered beam shifts in wavelength with a distribution [1, 2]. For each scattering angle φ, the scattered beam tends to have two peaks in the intensity spectrum. The wavelength of one peak is identical to the incident one, while the second peak shifts to a longer wavelength. Further, it has been found that this shift depends on the scattering angle. Quantitatively, the Compton shift in wavelength is given by [1-3]

$$\Delta\lambda/\lambda_C = 1 - \cos\varphi, \tag{1}$$

where $\Delta\lambda = \lambda_s - \lambda_i$, λ_i and λ_s are the wavelengths of the incident and the scattered beam, $\lambda_C = h/m_0 c$ called the Compton wavelength, m_0 the rest mass of the electron, and h Planck's constant (see Fig. 1). It is seen that the shift increases with the scattering angle to a maximum of $2\lambda_C$. The Compton wavelength of a free electron is $\lambda_C = 2.43 \times 10^{-12}$ m. In order for the Compton shift to be appreciable, the incident wavelength should not be much longer than λ_C. In Compton's experiment the wavelength of x rays is about 70 pm.
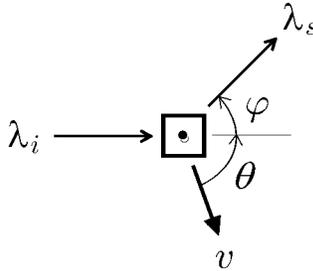

Fig. 1 The Compton effect. The dot represents an electron loosely bound in a target, which moves with speed v and angle θ after the collision.

In order to analyze this shift in wavelength, Compton adopted the collision between a photon and an electron. By taking hc/λ and h/λ as the energy and momentum of the photon according to the postulates of Einstein, the conservation of energy and momentum results in the following relations

$$\begin{cases} hc/\lambda_i = hc/\lambda_s + m_0 c^2(\gamma - 1) \\ h/\lambda_i = (h/\lambda_s)\cos\varphi + \gamma m_0 v \cos\theta \\ 0 = (h/\lambda_s)\sin\varphi - \gamma m_0 v \sin\theta, \end{cases} \tag{2}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$, v is the speed of the electron after the collision, and θ the angle from the recoiling electron to the incident beam. Then some algebra leads to the Compton

shift given by (1). As the collision between two particles in conjunction with the conservation of energy and momentum is used, the Compton effect is then commonly cited as evidence for the corpuscular picture of light.

3. Klein-Gordan Equation and Postulates of de Broglie

The Klein-Gordan equation proposed to govern the wavefunction Ψ of a free particle is a nonhomogeneous wave equation given by [6]

$$\left\{ \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right\} \Psi(\mathbf{r}, t) = \left(\frac{m_0 c}{\hbar}\right)^2 \Psi(\mathbf{r}, t), \tag{3}$$

where m_0 is the rest mass of the particle. Suppose that the wavefunction Ψ is a wave packet composed of plane waves with a narrow bandwidth. Each component of the plane waves is of the form of a space-time harmonic like $e^{ikx}e^{-i\omega t}$, where ω is the angular frequency and k the propagation constant. Then, for each of the plane waves, the wave equation reduces to an algebraic equation. That is,

$$\omega^2 - c^2 k^2 = \left(\frac{m_0 c}{\hbar}\right)^2. \tag{4}$$

It is seen that the relationship between ω and k is nonlinear and hence the matter wave is dispersive.

It is known that the peak of a wave packet moves at its group velocity. Thus the speed v of a particle can be given by the group speed v_g of the associated wave packet, that is, $v = v_g = d\omega/dk$. Then, from the preceding dispersion relation, one has

$$k = \frac{\omega}{c^2}v. \tag{5}$$

On substituting this relation back into the dispersion relation, one immediately has

$$\hbar\omega = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} = mc^2 \tag{6}$$

and then

$$\hbar k = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = mv, \tag{7}$$

where the speed-dependent mass

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}. \tag{8}$$

The preceding three formulas are just the postulates of de Broglie and the Lorentz mass variation law. Thus they can be viewed as consequences of the dispersion of matter wave.

By using a similar wave equation, which is based on the local-ether model of wave propagation and from which a first-order time evolution equation similar to Schrödinger's equation can be derived and thereby the particle velocity is evaluated in a quantum-mechanical way as the time derivative of expectation value of the position operator, the preceding five formulas have been given alternatively [7, 8]. Meanwhile, one fundamental difference is that the particle velocity determining the mass, energy, and momentum is referred to an earth-centered

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inertial frame for earthbound phenomena. However, as the linear speed due to earth's rotation is relatively low, it makes no substantial difference if the particle speed is referred instead to a geostationary laboratory frame, as done tacitly in common practice with the Compton scattering.

4. Wave Interpretation

We then go on to present the wave interpretation of the Compton effect, based on the interaction between electromagnetic and matter waves and on the scattering of electromagnetic waves. Under the illumination of an electromagnetic wave, the electrons loosely bound in the target tend to be accelerated by the incident electric and magnetic fields. Suppose that the electrons gain a velocity $\mathbf{v}$ from the action. Thus the initial and the final state of the electrons are respectively of $(m_0c^2, 0)$ and $(mc^2, m\mathbf{v})$ in energy and momentum, which in turn are represented by the wavefunctions Ψ_1 and Ψ_2 that incorporate the space-time harmonics $e^{-i\omega_0 t}$ and $e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})}$, respectively, where $\omega_0 = m_0c^2/\hbar$, $\omega = mc^2/\hbar$, and $\mathbf{k} = m\mathbf{v}/\hbar$. During a transition from the initial state to the final state, they are expected to form a mixed state $\Psi = c_1(t)\Psi_1 + c_2(t)\Psi_2$ with suitable coefficients c_1 and c_2. As the density of electrons is proportional to the product $\Psi^*\Psi$, this mixed state leads to a density incorporating a component that varies with space and time in the form

$$e^{i(\Delta\omega t - \Delta\mathbf{k}\cdot\mathbf{r})} = e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}e^{-i\omega_0 t}, \tag{9}$$

where $\Delta\omega = \omega - \omega_0$ and $\Delta\mathbf{k} = \mathbf{k}$, and its complex conjugate as well. It is noted that the temporal variation with $\Delta\omega$ is due to the mass variation. The mixed state results in time-varying charges and currents, which in turn radiate electromagnetic waves at the angular frequency $\Delta\omega$. This corresponds to the celebrated postulate of Bohr for the emission due to state transition of the electron bound in an excited atom or of high-energy electrons traveling in a synchrotron. Besides, the mixed state makes the permittivity of a dielectric medium vary by incorporating the space-time harmonic $e^{i(\Delta\omega t - \Delta\mathbf{k}\cdot\mathbf{r})}$, since the electric susceptibility of a medium is proportional to the charge density. (The complex conjugate term has a tendency to make the corresponding scattered wave have a higher frequency. This inverse Compton scattering [2] is omitted, as the electrons are initially stationary.)

It is known that the polarization current induced in a dielectric medium is determined by the product of the susceptibility and the electric field. Thus the induced charge and current in the space- and time-varying medium tend to incorporate a key component of which the spatial and temporal variation is given by the product of the ones of the incident field and the medium. That is,

$$e^{-i(\omega_s t - \mathbf{k}_s\cdot\mathbf{r})} = e^{-i(\omega_i t - \mathbf{k}_i\cdot\mathbf{r})}e^{i(\Delta\omega t - \Delta\mathbf{k}\cdot\mathbf{r})}, \tag{10}$$

where $e^{-i(\omega_i t - \mathbf{k}_i\cdot\mathbf{r})}$ and $e^{-i(\omega_s t - \mathbf{k}_s\cdot\mathbf{r})}$ denote the space- and time-variation of the incident wave and of the induced polarization, respectively. Thereby, $\omega_s = \omega_i - \Delta\omega$ and $\mathbf{k}_s = \mathbf{k}_i - \Delta\mathbf{k}$. The induced polarization in turn re-radiates scattered waves at the angular frequency ω_s in any possible direction. Thus the frequency and hence the wavelength of the scattered wave are changed, as a consequence of the interaction between electromagnetic and matter waves. Then we discuss its propagation direction, as a consequence of the scattering of electromagnetic waves.

A distribution of radiating currents with phase shift given by the space-harmonic $e^{i\mathbf{k}_s\cdot\mathbf{r}}$ behaves like a linear antenna array with the repeat distance of the antenna element being vanishing, as far as the radiation pattern is concerned. The progressive phase shift given by k_s determines the pattern. When $k_s \leq \omega_s/c$, a main beam with a strong radiation intensity due to constructive interference can form. Further, k_s determines the propagation direction of the main beam in such a way that the contributions from the various current elements

are in phase along that direction. As in an electronically steered phased-array radar, every direction is possible. Under the condition

$$k_s = \omega_s/c, \tag{11}$$

the main beam propagates just in the direction of $\mathbf{k}_s$, as in an end-fire antenna array [9].

Meanwhile, for a current distribution with different ω_s or $\mathbf{k}_s$ (in direction or magnitude), the scattered wave can also propagate in the aforementioned direction. However, the radiation is not of the main beam or the corresponding main beam is much narrower in terms of angular width, especially for an array much longer than the wavelength [9]. In addition to the space- and time-varying component given by (9), the permittivity of the target still has a major part which is invariant in space and time. This component in turn leads to scattered waves of which the frequency and wavelength remain unchanged from those of the incident wave. Furthermore, in addition to the re-radiation due to the polarization current, the mixed state also leads to a direct radiation due to the source varying with $e^{-i(\Delta\omega t - \Delta\mathbf{k}\cdot\mathbf{r})}$, as in the synchrotron radiation. This radiation has the angular frequency $\Delta\omega$ and hence the wavelength

$$\lambda_{rad} = \lambda_C/(\gamma - 1) = \lambda_i\lambda_s/\Delta\lambda. \tag{12}$$

However, its intensity does not depend strongly on the scattering angle and is expected to be low, since $|\Delta\mathbf{k}| \gg \Delta\omega/c$ as $\lambda_i \gg \lambda_C$ and thus the main beam disappears. This wavelength is inversely proportional to the mass variation as in the synchrotron radiation, while the Compton shift is approximately proportional to the variation. It tends to be well separated from λ_i and λ_s. Thus the prediction of the direct radiation may provide a means to demonstrate the mass variation and to test the wave interpretation.

Aside from the unchanged component of wavelength λ_i, the wave propagating in the direction of $\mathbf{k}_s$ is then mainly the scattered wave due to the induced current of which the space- and time-variation is given by $e^{-i(\omega_s t - \mathbf{k}_s\cdot\mathbf{r})}$, where $\omega_s = \omega_i - \Delta\omega$ and $\mathbf{k}_s = \mathbf{k}_i - \mathbf{k}$, subject to the phase condition $k_s = \omega_s/c$. Thereby,

$$k_i^2 + k^2 - 2k_i k \cos\theta = (\omega_i - \Delta\omega)^2/c^2, \tag{13}$$

where θ is the angle from $\mathbf{k}$ to $\mathbf{k}_i$. The use of the dispersion relation (4) and a little algebra leads to

$$k_i k \cos\theta = (\omega_i - \omega_s)(\omega_i + \omega_0)/c^2. \tag{14}$$

Then one has

$$k_i k_s \cos\varphi = k_i^2 - (k_i - k_s)(k_i + \omega_0/c), \tag{15}$$

where $k_s \cos\varphi = k_i - k\cos\theta$ is used. It is easy to show that this relation is identical to (1). Thus the Compton effect can be interpreted in an entirely different way. Based on the wave interpretation, the Compton scattering can be viewed as a demonstration of the wave nature of electrons, of the postulates of de Broglie, and of the wave equation, instead of a demonstration of the corpuscular picture of light.

In the Bragg diffraction of x rays from a crystal, the Bragg angle of reflection which corresponds to the constructive interference is determined by the path difference between waves reflected from two consecutive lattice planes of a certain spacing [3]. Alternatively, the lattices can be viewed as a medium of which the permittivity is time-invariant but is space-varying determined by the lattice constant. Thus the propagation vector of the reflected wave is changed depending on this constant, as depicted in Fig. 2a, and then the Bragg angle is determined by whether the phase condition (11) is fulfilled. On the other hand, for an electromagnetic wave incident upon a time-varying medium, the scattered wave tends to increase or decrease in frequency. This frequency shift has been observed experimentally for a microwave propagating in a rapidly growing plasma [10]. Similarly, the Raman effect is associated with the scattering from a sample where the atoms or molecules are subject to

periodic vibration or rotation [2]. The Raman scattering can also be observed with a crystal where a lattice wave due to atomic vibration (known as a phonon) is involved [11]. A sample with a lattice wave can be viewed as a medium of which the permittivity is space- and time-varying. Thus, as in the Compton scattering, both the frequency and the propagation vector of the reflected wave are changed by those of the lattice wave, as depicted in Fig. 2b. It is noted that the preceding interpretations are presented without explicit resort to the conservation laws for energy and momentum. Thus the conservation of energy and momentum in these phenomena can be viewed as a consequence of the scattering.
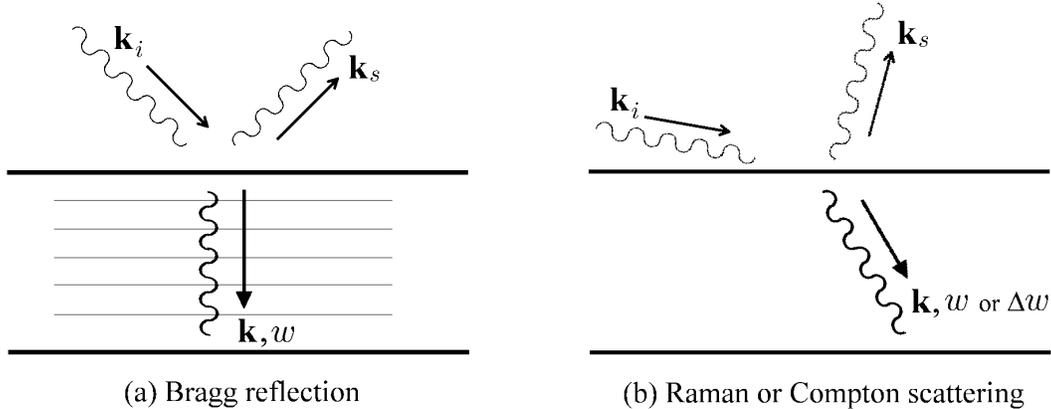

(a) Bragg reflection   (b) Raman or Compton scattering

Fig. 2 Diagrams for the propagation vectors in (a) the Bragg reflection and (b) the Raman or Compton scattering. $\mathbf{k}_s = \mathbf{k}_i - \mathbf{k}$. In the Bragg reflection $\mathbf{k}$ is determined by the lattice spacing and $\omega = 0$; in the Raman scattering $\mathbf{k}$ and ω are determined by the lattice wave; and in the Compton scattering $\mathbf{k}$ and $\Delta\omega$ are determined by the mixed state of matter wave.

5. Conclusion

Based on the postulates of de Broglie, which in turn are derived from the dispersion relation for matter wave, an entirely different interpretation is presented to account for the Compton effect. This approach deals with the interaction between electromagnetic and matter waves and with the constructive interference of electromagnetic waves scattered from a space- and time-varying medium. In this wave interpretation of the Compton scattering as well as those of the Bragg reflection and the Raman scattering, the conservation of energy and momentum is not used explicitly and thus it can be viewed simply as a consequence of the scattering. The temporal and spatial variation of the target in the Compton scattering in turn is due to the mixed state of matter wave of interacting electrons during their state transition and the temporal variation is a direct consequence of the mass variation. Thus the Compton scattering can be viewed as a demonstration of the postulates of de Broglie for high-speed particles and of the wave equation. In addition to the scattered wave, a weak radiation of which the wavelength is much longer than the one of the scattered wave is predicted. Its wavelength depends directly on the mass variation as in the synchrotron radiation. This prediction may provide a means to test the wave interpretation.

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