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Reinterpretation of the effects of the Earth's rotation and gravity on the neutron-interferometry experiment

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Abstract. – The phase shift of matter wave due to the Earth's rotation and gravity has been demonstrated in the neutron-interferometry experiments where the Bragg reflection of the neutron wave from crystals is used to form a closed path. In the literature, the rotation-induced phase shift is ascribed to an extra energy term associated with angular momentum in the Hamiltonian. In this investigation, based on a classical treatment of the collision of neutrons with the lattice planes, the modifications in the particle velocity due to the Earth's rotation as well as those due to gravity are calculated. Then, in conjunction with the relation between the particle velocity and the propagation vector of matter wave, it is shown that the corresponding phase shifts in matter wave are in accord with the interference experiments. Thereby, based on the velocity modification due to collision, a reinterpretation of the effects of the Earth's rotation and gravity on the neutron-wave interference is presented. Meanwhile, a null effect of the Earth's orbital motion is predicted.

Introduction. – The wave nature of particles has been initiated by de Broglie by postulating that a particle is associated with a matter wave the wavelength of which is related to the momentum of the particle. The matter wavelength was first demonstrated by Davisson and Germer in the Bragg reflection of an electron beam from a crystal. More recently, the neutron-wave loop interferometry has been achieved, where two matter waves are coherently split from a particle beam, propagate, respectively, along separated paths, and then are combined to cause an interference depending on the phase difference between them. By using the loop interferometry, the effects of the Earth's rotation and gravity on the phase shift of a neutron beam have been demonstrated with high precision [1,2]. The rotation-induced phase shift is accounted for by introducing an extra interaction term of angular momentum in the Hamiltonian, in addition to a term of gravitational potential energy. Thus, the canonical momentum, to which the matter wavelength is supposed to be related, incorporates a term associated with the Earth's rotation [1,2]. In this way the effect of the Earth's rotation on the matter wave, like that of the gravitational potential, is supposed tacitly to be a function of space during the particle's travelling along the paths.

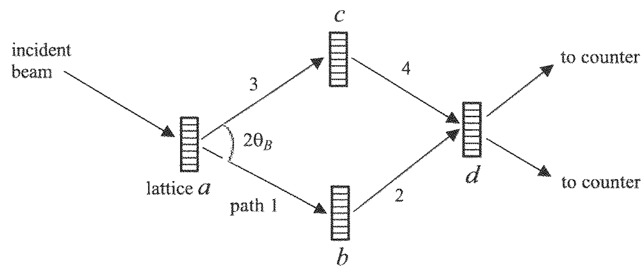

Fig. 1 – Diagram of the neutron-wave interferometer formed by four slabs of silicon crystal for the Bragg reflection.

Recently, we have presented the local-ether model of wave propagation whereby the electromagnetic wave is supposed to propagate via a medium like the ether [3]. However, the ether is not universal. It is supposed that in the region under sufficient influence of the gravity due to the Earth, the Sun, or another celestial body, a local ether forms which in turn moves with the gravitational potential of the respective body. For earthbound waves the medium is the Earth local ether which, as well as the Earth's gravitational potential, is stationary in an ECI (Earth-centered inertial) frame, while the Sun local ether for interplanetary waves is stationary in a heliocentric inertial frame. The local-ether model has been used to account for the effects of the Earth's motions in a wide variety of propagation phenomena of electromagnetic wave, including the famous Michelson-Morley experiment and the Sagnac effect in loop interferometry [3]. Furthermore, matter wave is supposed to follow the local-ether propagation model and is governed by a wave equation incorporating the gravitational and the electrical scalar potentials [4,5]. Under the ordinary condition of low particle speed, the local-ether wave equation has been shown to lead to a unified quantum theory of gravitational and electromagnetic forces [4]. Furthermore, it leads to the important consequence that the gravitational mass associated with the gravitational force is identical to the inertial mass under the influence of the electromagnetic force. Moreover, due to the Earth's rotation, it leads to the east-west directional anisotropy in the speed-dependent mass, the quantum state energy, and hence in the clock rate which in turn has been demonstrated in the Hafele-Keating experiment with circumnavigation atomic clocks [5]. Meanwhile, earthbound experiments associated with matter or electromagnetic wave are entirely independent of the Earth's orbital motion.

For a propagating harmonic-like wave function, the local-ether wave equation leads to the relation $\boldsymbol{k} = m\boldsymbol{v}/\hbar$, where $\boldsymbol{k}$ is the propagation vector of the matter wave associated with a particle of mass m, and $\boldsymbol{v}$ is the velocity of the particle referred specifically to the local-ether frame [5]. This relation looks like the famous postulate of de Broglie, except the reference frame of the particle velocity. In this investigation, based on a classical treatment of the collision of neutrons with the lattice planes, the modifications in the particle velocity due to the rotation as well as those due to gravity are calculated. Then the influence of the velocity modification on the matter wave is applied in a consistent way to account for the loop-interferometry experiments which demonstrate the effects of the Earth's rotation and gravity on neutron wave.

Loop interferometry and effect of the Earth's gravity. – Consider the loop interferometry of neutron wave discussed in [1,2], where a closed path is formed by using the Bragg reflection from four slabs of silicon crystal. Part of the incident neutron beam is transmitted through a silicon slab (lattice a) to travel along path 1 (fig. 1). Meanwhile, the other part is reflected by lattice a to travel along a different path 3. The angle between paths 1 and 3 is $2\theta_B$, where the Bragg angle θ_B is determined by the constructive interference between the waves reflected from two consecutive lattice planes. Thus the incident beam is split into two coherent beams along two separated paths. Thereafter, the two beams are reflected by lattices b and c, respectively, and then are combined by using another lattice d to reflect parts of the beams. The intensity of the combined beam is measured by a counter to determine the phase difference between the beam along paths 1 and 2 and the one along paths 3 and 4. The four silicon slabs are parallel and the four paths of directed length $\boldsymbol{l}_i$ form a closed path of a parallelogram shape, where the index $i = 1, 2, 3, 4$.

Suppose the velocity of the neutron beam in path i is $\boldsymbol{v}_i$ and the velocity of the laboratory frame in which the lattices are stationary is $\boldsymbol{v}_0$, all referred to the local-ether frame which is an ECI frame for the earthbound experiment. For a geostationary laboratory, the velocity $\boldsymbol{v}_0$ is due to the Earth's rotation alone. Then, from classical mechanics, it is known that the collision of a particle with the rigid planes leads to $\boldsymbol{v}_4 = \boldsymbol{v}_1$ and

$$\boldsymbol{v}_2 = \boldsymbol{v}_3 = \boldsymbol{v}_1 - 2(\boldsymbol{v}_1 - \boldsymbol{v}_0) \cdot \hat{n}\hat{n}, \tag{1}$$

where $\hat{n}$ $(= (\hat{l}_2 - \hat{l}_1)/\sqrt{2 - 2\hat{l}_2 \cdot \hat{l}_1})$ denotes the normal of the reflecting lattice planes. It is easy to show that the particle speed v_p $(= |\boldsymbol{v}_i - \boldsymbol{v}_0|)$ with respect to the laboratory frame is identical in the four paths. Furtherermore, it is supposed that the phase variation ϕ of matter wave over a path of directed length $\boldsymbol{l}$ along the particle beam is given by $\phi = \boldsymbol{k} \cdot \boldsymbol{l}$. This expression looks like that adopted in [1, 2], except some extra terms in the propagation vector $\boldsymbol{k}$, and the reference frame of the particle velocity $\boldsymbol{v}_i$ associated with $\boldsymbol{k}$. Thus the phase difference between the two coherent neutron beams along the paths is given by

$$\Delta\phi\hbar/m_0 = -\boldsymbol{v}_1 \cdot \boldsymbol{l}_1 - \boldsymbol{v}_2 \cdot \boldsymbol{l}_2 + \boldsymbol{v}_3 \cdot \boldsymbol{l}_3 + \boldsymbol{v}_4 \cdot \boldsymbol{l}_4, \tag{2}$$

where the particle speed v_i is assumed to be much lower than c and hence the speed-dependent mass m is substantially equal to the rest mass m_0. The direction of $\boldsymbol{v}_i$ in general is different from that of $\boldsymbol{l}_i$. Anyway, due to the symmetry in the particle velocities ($\boldsymbol{v}_1 = \boldsymbol{v}_4$ and $\boldsymbol{v}_2 = \boldsymbol{v}_3$) and in the paths ($\boldsymbol{l}_1 = \boldsymbol{l}_4$ and $\boldsymbol{l}_2 = \boldsymbol{l}_3$), the phase difference becomes zero.

We then proceed to consider the case where the gravity is taken into account. Due to the gravity $\boldsymbol{g}$, the particle velocity $\boldsymbol{v}_i$ tends to increase by an amount $\boldsymbol{g}t$, where t is the time interval after the particle enters path i. Here we have made use of the identity of gravitational and inertial mass, a result derived in [4]. Thus, over path 1 or 4 and over path 2 or 3, this gravity term results in a phase variation (multiplied by $\hbar/m_0$) of $\boldsymbol{l}_1 \cdot \boldsymbol{g}\tau_1/2$ and $\boldsymbol{l}_2 \cdot \boldsymbol{g}\tau_2/2$, respectively, where the traveling time τ_i $(= l_i/v_p)$ over path i is determined by the particle speed referred to the laboratory frame. Furthermore, due to the collision with lattice b, the particle velocity $\boldsymbol{v}_2$ increases by an amount $\tau_1(\boldsymbol{g} - 2\boldsymbol{g} \cdot \hat{n}\hat{n})$ immediately after being reflected from lattice b. Thus, over path 2, the total contribution to the phase variation due to gravity is the sum $\boldsymbol{g} \cdot (\hat{l}_1 l_2 \tau_1 + \boldsymbol{l}_2\tau_2/2)$, where the first term is associated with the collision and is a result of the identity $(\boldsymbol{g} - 2\boldsymbol{g} \cdot \hat{n}\hat{n}) \cdot \hat{l}_2 = \boldsymbol{g} \cdot \hat{l}_1$. Similarly, by taking the collision with lattice c into account, the phase variation over path 4 due to gravity is the sum $\boldsymbol{g} \cdot (\hat{l}_2 l_1 \tau_2 + \boldsymbol{l}_1\tau_1/2)$. Thus the gravitation-induced phase difference becomes

$$\Delta\phi\hbar/m_0 = \boldsymbol{g} \cdot (\hat{l}_2 - \hat{l}_1) l_2 l_1/v_p. \tag{3}$$

It is noted that some of the terms due to gravity cancel out and the surviving terms in the phase difference are associated with the collision of the particle with the lattice planes. It can be shown that if the collision were not taken into account, the preceding formula would happen to change its sign. It will be shown later on that this formula leads to a gravitation-induced phase difference identical to the one given in [1,2] which in turn is derived from an alternative approach based on the Hamiltonian and has been demonstrated experimentally. It is of interest to note that the preceding formula of matter-wave interference is independent of the laboratory velocity $\boldsymbol{v}_0$. This is a consequence of the situation that the paths in the interferometer form a closed loop. Thus, in spite of the restriction on the reference frame of the particle velocity $\boldsymbol{v}_i$, the gravitation-induced quantum interference complies with Galilean relativity.

Effect of the Earth's rotation on interference. – Precisely, owing to the Earth's rotation and the various distances from the Earth's center, the velocities of the reflecting lattices tend to be different among themselves. Thus the lattice velocity $\boldsymbol{v}_{\rm l}$ with respect to an ECI frame is not exactly equal to the laboratory velocity $\boldsymbol{v}_0$, but is given by $\boldsymbol{v}_{\rm l} = \boldsymbol{v}_0 + \bar{\omega}_{\rm E} \times (\boldsymbol{r} - \boldsymbol{r}_0)$, where $\boldsymbol{r}$ and $\boldsymbol{r}_0$ denote the position vectors of the lattice and a suitable reference point at which the laboratory velocity is defined, respectively, and $\bar{\omega}_{\rm E}$ is the directed rate of the Earth's rotation. Hence the particle velocities $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ given in (1) together with $\boldsymbol{v}_4$ need some modifications. From classical mechanics, it is seen that due to the collision of particles with the rigid lattices b and a, the velocities $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ are given by (1) with $\boldsymbol{v}_0$ being replaced by the lattice velocities $\boldsymbol{v}_{{\rm l}b}$ and $\boldsymbol{v}_{{\rm l}a}$, respectively. Furthermore, it can be shown that due to the collision with lattice a and then lattice c, the velocity $\boldsymbol{v}_4$ is given by

$$\boldsymbol{v}_4 = \boldsymbol{v}_1 - 2(\boldsymbol{v}_{{\rm l}a} - \boldsymbol{v}_{{\rm l}c}) \cdot \hat{n}\hat{n}, \tag{4}$$

where $\hat{n}$ is the normal given previously and the gravity is neglected for simplicity.

Then, due to the symmetry in the paths, the phase difference (2) becomes

$$\Delta\phi\hbar/m_0 = -2(\boldsymbol{v}_{{\rm l}a} - \boldsymbol{v}_{{\rm l}c}) \cdot \hat{n}\hat{n} \cdot \boldsymbol{l}_1 + 2(\boldsymbol{v}_{{\rm l}a} - \boldsymbol{v}_{{\rm l}b}) \cdot \hat{n}\hat{n} \cdot \boldsymbol{l}_2. \tag{5}$$

The velocity differences between the various lattices can be given by $\boldsymbol{v}_{{\rm l}a} - \boldsymbol{v}_{{\rm l}b} = -\bar{\omega}_{\rm E} \times \boldsymbol{l}_1$ and $\boldsymbol{v}_{{\rm l}a} - \boldsymbol{v}_{{\rm l}c} = -\bar{\omega}_{\rm E} \times \boldsymbol{l}_2$. By expressing the normal $\hat{n}$ in terms of $\hat{l}_1$ and $\hat{l}_2$, it is easy to show that the rotation-induced phase difference becomes

$$\Delta\phi\hbar/m_0 = 2\bar{\omega}_{\rm E} \cdot \boldsymbol{S}, \tag{6}$$

where $\boldsymbol{S}$ $(= \boldsymbol{l}_2 \times \boldsymbol{l}_1)$ is the directed area of the loop formed by the paths and its magnitude $S = l_1 l_2 \sin 2\theta_{\rm B}$. This rotation-induced phase difference is identical to the one given in [1,2] which is derived from the Hamiltonian approach and has been demonstrated experimentally. Again, it is seen that the preceding formula is independent of the laboratory velocity $\boldsymbol{v}_0$. Thus, in spite of the restriction on the reference frame of the particle velocity, the rotation-induced quantum interference also complies with Galilean relativity. The preceding formula is known to exhibit the matter-wave Sagnac effect, as it is similar to the one of the Sagnac effect of an electromagnetic wave caused by the shift of the wave propagation time due to the rotation of the receiver with respect to the local-ether frame [3].

Specifically, consider the situation where paths 1 and 4 are horizontal, as that discussed in [1,2]. The loop is rotated about a horizontal axis accommodating path 1 by a rotation angle α, where $\alpha = 0$ corresponds to the loop lying on a horizontal plane. The rotation axis is parallel to $\boldsymbol{l}_1$ or to the incident beam directed due south, such that path 4 is raised to a higher position than path 1 when $0 < \alpha < \pi$. Thus $\boldsymbol{g} \cdot \hat{l}_1 = 0$ and $\boldsymbol{g} \cdot \hat{l}_2 = -g \sin 2\theta_{\rm B} \sin\alpha$. The phase difference due to the Earth's rotation and gravity becomes

$$\Delta\phi\frac{\hbar}{m_0} = -2\omega_{\rm E} S \cos\theta_{\rm L} \cos\alpha - \frac{g}{v_{\rm p}} S \sin\alpha, \tag{7}$$

where $\theta_{\rm L}$ is the colatitude angle. This formula is identical to that given in [2]. Consider another specific situation where paths 1 and 4 are vertical along a plumb line with the average altitude of path 4 being higher than that of path 1, as that discussed in [1]. The loop is rotated about a vertical axis accommodating path 1 by a rotation angle α, where $\alpha = 0$ corresponds to the loop facing due west. The rotation axis is antiparallel to $\boldsymbol{l}_1$ or to the incident beam directed upward, such that the loop faces due north or south when $\alpha = \pi/2$ or $3\pi/2$, respectively, and then the directions comply with fig. 21 in [1]. Thus $\boldsymbol{g} \cdot \hat{l}_1 = -g$ and $\boldsymbol{g} \cdot \hat{l}_2 = -g \cos 2\theta_{\rm B}$. The phase difference due to the Earth's rotation and gravity becomes

$$\Delta\phi \frac{\hbar}{m_0} = 2\omega_{\rm E} S \sin\theta_{\rm L} \sin\alpha + \frac{g}{v_{\rm p}} S \tan\theta_{\rm B}. \tag{8}$$

It is seen that the phase difference due to gravity is independent of α. Thus, as the loop is rotating around the axis with a changing α, the variation in the phase difference is due to the Earth's rotation alone.

Comparison between different approaches. – The derivation of neutron interferometry adopted in [1, 2] is based on the Hamiltonian which incorporates the kinetic energy with the inertial mass, the gravitational energy with the gravitational mass, and an additional energy term associated with the Earth's rotation rate and angular momentum. Then, from Hamilton's equations, the canonical momentum is derived, which incorporates the product $\boldsymbol{g}t$ discussed previously, the linear velocity due to the Earth's rotation, and other minor terms. By resorting to the principle of equivalence, the inertial mass is equal to the gravitational mass. Then the gravitation- and rotation-induced quantum interferences are derived which agree with those given by (3) and (6) based on the collision of particles with the lattices. Another approach, somewhat similar to the presented one, has been given for the rotation-induced phase difference, where the Doppler effect on the propagation vectors of the waves reflected from moving lattices is used in the derivation for matter or optical wave [6]. Besides these, there are some other approaches for the derivation of the matter-wave Sagnac effect, as discussed in [1].

In a path integral for evaluating the gravitation-induced phase difference in the Hamiltonian approach, the gravity-induced contribution to the canonical momentum is given without explanation to be parallel to the direction of the path (see eq. (11) in [2]) rather than simply to the gravity itself (eqs. (24) and (28) in [1]). Thus some kind of collision has been assumed tacitly all the way along the path such that the gravity-induced contribution to the particle velocity will be deflected to follow the path direction. Another difference is that in the Hamiltonian approach for the rotation-induced phase difference, it has been supposed tacitly that the effect of the Earth's rotation on the matter wave is a function of space and hence the propagation vector tends to change continuously during the particle's traveling along the paths. On the other hand, in our collision approach or in [6], the Earth's rotation changes the property of matter wave merely at the instant when the particle collides with the lattice and the propagation vector as well as the particle velocity remain unchanged over the traveling along each individual path.

Furthermore, a fundamental difference is the effect of the Earth's orbital motion around the Sun or others. This effect is not mentioned in the Hamiltonian approach or others and the energy associated with this motion is not included in the Hamiltonian in a way analogous to the one with the Earth's rotation. It is noted that this omitted energy is much higher than the one with the Earth's rotation, although the associated Sagnac effect (if it does exist) is much smaller by a factor of about 1/365. This effect might be too small to be measured. Actually, no Sagnac interference fringe for either matter or electromagnetic wave has been reported in the literature, to our knowledge. In some earthbound experiments with electromagnetic waves, this effect has been assumed to be null by resorting to the principle of local Lorentz invariance (see [3] for a discussion). Anyway, according to the local-ether propagation model, earthbound experiments can depend on the Earth's rotation but are entirely independent of the Earth's orbital motion around the Sun or whatever. Thus the particle velocity which determines the propagation vector $\boldsymbol{k}$ of matter wave is referred uniquely to an ECI frame, rather than to a heliocentric inertial, the ECEF (Earth-centered Earth-fixed), or any other frame, while the phase difference in a loop path can be invariant in a frame moving at a fixed velocity in an ECI frame. Thereby, the local-ether model unambiguously predicts a discrepancy between the effects of the rotational and the orbital motions of the Earth in the Sagnac loop interferometry as well as in many other earthbound experiments discussed in [3, 5], which provides a means to test its validity.

Conclusion. – The phase shift in the neutron-interferometry experiments due to the Earth's rotation as well as that due to gravity is accounted for simply by a classical treatment of the collision of particles with rigid planes in conjunction with the propagation vector of matter wave. Owing to the symmetry in the particle velocities and in the paths, the phase difference tends to be zero. However, the velocity symmetry breaks down if the Earth's rotation or gravity is taken into account. From the collision of neutrons with the lattice planes, the velocity modifications due to the rotation and gravity are derived and the resulting phase differences agree with those observed in the neutron-interferometry experiments. According to the local-ether model, the particle velocity which determines the matter wavelength is referred uniquely to an ECI frame. However, the phase difference in a loop interferometer can be independent of the laboratory velocity and hence comply with Galilean relativity. Moreover, it is predicted that the Earth's orbital motion has no effects on the earthbound interferometry. The discrepancy between the effects of the Earth's rotational and orbital motions provides a means to test the local-ether wave equation.

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