Reinterpretation of Matter-Wave Interference Experiments Based on the Local-Ether Wave Equation

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Abstract – Based on the local-ether wave equation for free particle, the dispersion of matter wave is examined. From the dispersion relation, the angular frequency and wavelength of matter wave are derived. These formulas look like the postulates of de Broglie in conjunction with the Lorentz mass-variation law. However, the fundamental difference is that for terrestrial particles their speeds are referred specifically to a geocentric inertial frame and hence incorporate the speed due to earth's rotation. Thus the local-ether model predicts an east-west directional anisotropy both in mass and wavelength. Meanwhile, in spite of the restriction on reference frame, the local-ether model can account for the matter-wave interference experiments of the Bragg reflection and the Sagnac effect. For electron wave, the effects of earth's rotation are negligible and the derived Bragg angle is actually in accord with the Davisson-Germer experiment, as examined within the present precision. On the other hand, the local-ether model leads to a directional anisotropy in the Bragg angle in neutron diffraction. The predicted anisotropy due to earth's rotation then provide a means to test the local-ether wave equation.

1. Introduction

The wave nature of particle has been initiated by de Broglie in 1924 by postulating that a particle is associated with a matter wave of which the angular frequency and the wavelength are related to the energy and the momentum of the particle, respectively [1]. The energy and momentum in turn are associated with the speed-dependent mass which was first introduced by Lorentz in 1904 [2]. Shortly, the matter wavelength has been demonstrated in the Bragg reflection of electron beam from a crystal by Davisson and Germer in 1927 [1]. Furthermore, based on the phase difference between two coherent beams of electron, neutron, or atom, various matter-wave interference experiments have been reported to demonstrate the Bragg reflection, the double-slit diffraction, the gravitational effect, and the Sagnac effect.

Recently, we have presented the local-ether model of wave propagation [3]. That is, electromagnetic wave can be viewed as to propagate via a medium like the ether. However, the ether is not universal. It is supposed that in the region under sufficient influence of the gravity due to the Earth, the Sun, or another celestial body, there forms a local ether which in turn moves with the gravitational potential of the respective body. For earthbound waves, the medium is the earth local ether which as well as earth's gravitational potential is stationary in an ECI (earth-centered inertial) frame, while the sun local ether for interplanetary waves is stationary in a heliocentric inertial frame. It has been shown elaborately that the propagation of earthbound electromagnetic waves is referred specifically to an ECI frame, as demonstrated in the high-precision experiments of GPS (global positioning system), the intercontinental microwave link, and of the Sagnac loop interferometry [3].

Further, matter wave is supposed to follow the local-ether model. Thereby, we have presented a wave equation for a particle of charge q and natural frequency ω_0 in the presence of the gravitational and the electrical scalar potentials [4-6]. Under the ordinary condition of low particle speed, the local-ether wave equation has been shown to lead to a unified quantum theory of gravitational and electromagnetic forces [5, 6]. Furthermore, it is found that the gravitational mass associated with the gravitational force and the inertial mass under the influence of the electromagnetic force are identical to the natural frequency, aside from a common scaling factor. Thereby, the local-ether wave equation leads to the important

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consequence of the identity of gravitational and inertial mass and their physical origin. When the restriction on the particle speed is removed, the local-ether wave equation leads to the east-west directional anisotropy in mass, quantum state energy, and hence in clock rate which in turn has been demonstrated in the Hafele-Keating experiment with circumnavigation atomic clocks [4].

In this investigation, we explore more consequences of the local-ether wave equation. From the wave equation for a free particle, the dispersion of matter wave is derived. Then, from the dispersion relation, the angular frequency and wavelength of matter wave and the associated speed-dependent mass of particle are derived. According to the local-ether model, the velocity of earthbound particles that determines the mass and the wave properties is referred specifically to an ECI frame and hence incorporates the linear velocity due to earth's rotation. Accordingly, the matter-wave interference experiments of the Bragg reflection, the double-slit diffraction, and of the Sagnac effect are reexamined, particularly the effects of earth's rotational and orbital motions.

2. Local-Ether Wave Equation for Free Particle

It is supposed that a free particle is represented by a wavefunction Ψ which in turn is governed by the nonhomogeneous wave equation proposed to be

$$\left\{ \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right\} \Psi(\mathbf{r}, t) = \frac{\omega_0^2}{c^2}\Psi(\mathbf{r}, t),\tag{1}$$

where c is the speed of light and ω_0 is the natural frequency associated with the particle. This equation is a simplified form of the local-ether wave equation presented in [4-6] by dropping the gravitational and the electric scalar potentials. For the case where the natural frequency is zero, the equation reduces to the homogeneous wave equation governing electromagnetic wave in free space. Further, by replacing c with another speed, the equation reduces to the wave equation governing a mechanical wave.

The natural frequency is a constant and has been found to be the origin of the rest mass of the particle associated with wavefunction Ψ. It has been shown from the local-ether wave equation under the gravitational and the electric potential potentials that the natural frequency ω_0 is related to the gravitational and the inertial mass m_0 of a low-speed particle by the familiar form of [4-6]

$$m_0 = \frac{\hbar}{c^2}\omega_0,\tag{2}$$

where $\hbar$ is Planck's constant divided by 2π. This relation states that the rest mass of a particle is just its natural frequency, aside from the scaling factor $\hbar/c^2$.

Since matter wave is supposed to follow the local-ether model, the time derivative in the proposed wave equation is referred specifically to an ECI frame for earthbound particles. As the time derivative is referred to a particular frame, its value remains unchanged when observed in different reference frames. This feature is identical to that in a mechanical wave motion. The proposed wave equation looks like the free-space Klein-Gordan equation [7, 8] and like the equation dealing with spin given in [9], except the fundamental difference in the reference frame of the time derivative.

From the wave equation, it is seen that the wavefunction Ψ oscillates at the natural frequency when its spatial variations vanish. As the wavefunction starts to vary spatially, its temporal variation will increase accordingly. The spatial and the temporal variations of a wavefunction are important characteristics of wave motion and are expected to play an important role in determining the physical properties of the particle associated with the wavefunction.

3. Dispersion of Matter Wave

Suppose that the wavefunction Ψ is a wave packet composed of plane waves of a narrow bandwidth. Each component of the plane waves is of the form $\psi_0 e^{ikx}e^{-i\omega t}$, where ω is the

angular frequency, k the propagation constant, and ψ_0 an arbitrary constant. Then, for each of the plane waves, the wave equation leads to an algebraic equation

$$\omega^2 - c^2 k^2 = \omega_0^2. \tag{3}$$

It is seen that the angular frequency and the propagation constant are related to each other. Further, owing to the presence of the natural frequency ω_0, the relation between ω and k becomes nonlinear and hence the matter wave is dispersive. For electromagnetic wave with a zero natural frequency, the dispersion then vanishes. On the other hand, the dispersion relation is important in determining the properties of matter wave.

It is known that the peak of a wave packet moves at its group velocity. Thus the speed v of a particle can be given by the group speed v_g of the associated wave packet, that is, $v = v_g = d\omega/dk$. Then, from the preceding dispersion relation, one has $\omega d\omega = c^2 k dk$ which in turn yields the propagation constant-speed relation

$$k = \frac{\omega}{c^2} v. \tag{4}$$

It is noted that the propagation constant k is proportional to the speed v with the angular frequency ω as a ratio.

On substituting this relation back into the dispersion relation, it is seen that the frequency ω in turn depends on the speed v as

$$\omega = \frac{\omega_0}{\sqrt{1 - v^2/c^2}}. \tag{5}$$

In terms of the natural frequency ω_0, the propagation constant-speed relation becomes

$$k = \frac{\omega_0}{c^2} \frac{1}{\sqrt{1 - v^2/c^2}} v. \tag{6}$$

Since wavelength is inversely proportional to propagation constant, the wavelength of a harmonic-like matter wave packet also depends on the particle speed.

In terms of the rest mass m_0, the angular frequency and the propagation constant take the form of

$$\hbar\omega = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \tag{7}$$

and

$$\hbar k = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}, \tag{8}$$

respectively. It is of essence to note that the preceding two formulas look like the postulates of de Broglie. However, the fundamental difference is that the particle speed here is referred specifically to the local-ether frame, which is an ECI frame for earthbound particles. Ignoring this difference in reference frame, the famous postulates of de Broglie are then consequences derived from the dispersion of matter wave.

From the frequency-speed relation (5), one immediately has

$$v = c\sqrt{1 - \omega_0^2/\omega^2}. \tag{9}$$

It is seen that the particle speed v increases monotonically with the angular frequency ω. Further, the speed of a free particle is then limited to c, as referred to the local-ether frame. For this limiting case, the propagation constant k and hence the angular frequency ω are large enough, such that the natural frequency ω_0 in the dispersion relation (3) can be neglected. As a result, the propagation characteristics of such a matter wave are close to those of electromagnetic wave. For terrestrial high-energy electrons emitted spontaneously from radiative elements or accelerated in a synchrotron, the limiting speed is expected to be

referred to an ECI frame. However, as the linear speed due to earth's rotation is relatively low, it makes no substantial difference in measurement if the speed is referred instead to a geostationary laboratory frame, as done tacitly in common practice.

A dispersion relation quite similar to (3) can be found in the plane wave propagating in a plasma, with the natural frequency ω_0 being replaced with the plasma frequency [10]. Another similar dispersion can be found in the guided mode propagating in a waveguide, with the natural frequency ω_0 being replaced with the cutoff frequency [10]. Hence a frequency-speed relation similar to (5) or (9) can be found in a plasma or a waveguide.

4. Mass, Energy, and Momentum

The formulas of angular frequency and propagation constant can be written in a more compact way by introducing the speed-dependent mass m defined in terms of the rest mass m_0 in a familiar form of

$$m = m_0 \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{10}$$

Then we have the frequency-mass relation

$$\hbar\omega = mc^2 \tag{11}$$

and the propagation vector-velocity relation

$$\hbar\mathbf{k} = m\mathbf{v}. \tag{12}$$

This speed-dependent mass looks like the famous Lorentz mass-variation law. However, the fundamental difference is that the particle speed v in (10) is referred specifically to the local-ether frame, rather than a laboratory frame or any other. Thus the speeds of terrestrial particles incorporate the one due to earth's rotation, while the corresponding wave properties are entirely independent of earth's orbital motion around the Sun or others.

From the preceding relations, it is seen that the rest mass m_0 and the speed-dependent mass m of a particle are just the natural frequency and the angular frequency of the associated matter wave, respectively, aside from the common scaling factor $\hbar/c^2$. Thereby, based on the local-ether wave equation, the mass originates from the temporal variation of matter wave. And the mass variation originates from the dispersion of matter wave, which in turn is due to the natural frequency. On the other hand, it is noticed that the standard derivation of the mass variation is through a quite indirect way by dealing with a case associated with an elastic or inelastic collision between two identical particles [11-13].

For a high-speed terrestrial particle, it may make no substantial difference whether the particle speed is referred to an ECI frame or to the ground. Nevertheless, one consequence of the local-ether model is that the speed-dependent mass m as well as the matter wavelength is expected to possess an east-west directional anisotropy due to earth's rotation. That is, for a given ground speed, the earthbound particles moving eastward have the highest speed with respect to an ECI frame and hence have the heaviest mass, while those moving westward, the lightest. This anisotropy in mass in conjunction with the mass-dependence of quantum energy of the matter wave bounded in atoms has been adopted to account for the east-west directional anisotropy in the clock rate observed in the Hafele-Keating experiment with circumnavigation atomic clocks [4].

Similar to those in quantum mechanics, the *momentum* and the *energy* of a particle are defined as the expectation values of the spatial and the temporal derivatives, $\langle -i\hbar\nabla \rangle$ and $\langle i\hbar\partial/\partial t \rangle$, respectively, where the expectation value of an operator O is evaluated in terms of the wavefunction Ψ as $\langle O \rangle = \int \Psi^* O \Psi d\mathbf{r}$. Thus, for a space-harmonic wave, the momentum is given by $\mathbf{p} = \hbar\mathbf{k}$. Then, according to the propagation vector-velocity relation (12), the momentum becomes a familiar form of $\mathbf{p} = m\mathbf{v}$. However, the particle velocity $\mathbf{v}$ is referred specifically to the local-ether frame. From the local-ether wave equation, a first-order time evolution equation, similar to Schrödinger's equation, has been derived. From this equation it has been shown that the velocity of a particle defined as the time derivative

of expectation value of the position vector corresponds to the expectation value of a spatial derivative as $\mathbf{v} = \langle -i\hbar\nabla \rangle /m_0$, where the velocity is referred to the local-ether frame and the corresponding speed is supposed to be much lower than c [4-6]. It is of interesting to note that for a wave packet, the particle speed derived from the evolution equation is identical to the aforementioned group velocity derived from the dispersion relation.

Furthermore, for a time-harmonic wave, the energy is given by $\mathcal{E} = \hbar\omega$. Then, according to the frequency-mass relation (11), the energy becomes a familiar form of

$$\mathcal{E} = mc^2. \tag{13}$$

Ignoring the difference in reference frame of the particle speed, this is just the famous energy-mass relation first introduced by Poincaré in 1900 [14]. For a low-speed particle, the mass-variation law (10) leads to

$$\hbar\omega - \hbar\omega_0 = mc^2 - m_0c^2 = \frac{1}{2}m_0v^2. \tag{14}$$

Thus, due to the dispersion of matter wave, the variation in the angular frequency or in the mass corresponds to the *kinetic energy* for a low-speed particle. When the spatial variation of Ψ is weak, the temporal variation of Ψ is close to the harmonic $e^{-i\omega_0 t}$ and then the wavefunction can be given as $\Psi(\mathbf{r}, t) = \psi(\mathbf{r}, t)e^{-i\omega_0 t}$, where the temporal variation of the wavefunction ψ is weak. The aforementioned evolution equation derived from the wave equation is expressed in terms of this reduced wavefunction ψ. It is seen that the energy and the kinetic energy are associated with the expectation values of the time derivative $i\hbar\partial/\partial t$ evaluated in terms of wavefunctions Ψ and ψ, respectively.

As the particle velocity is referred to a specific frame, all the values of the angular frequency, the propagation vector, the energy, the kinetic energy, and of the momentum remain unchanged when observed in different reference frames. For earthbound particles, the momentum and the kinetic energy are then referred specifically to an ECI frame. This differs from the common understanding, since the conventional momentum and kinetic energy are not attached to a specific frame and hence are different in different frames. However, in what follows we show that the local-ether momentum and kinetic energy indeed comply with the conservation laws of the conventional momentum and kinetic energy.

Consider the collision between two low-speed particles of rest masses m_1 and m_2. It is supposed that the sum of the propagation constants and the one of the angular frequencies of the two particles remain fixed during the collision. Thus

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_3 + m_2\mathbf{v}_4 \tag{15}$$

and

$$m_1v_1^2 + m_2v_2^2 = m_1v_3^2 + m_2v_4^2, \tag{16}$$

where $\mathbf{v}_1$ and $\mathbf{v}_3$ are the velocities of the particle of m_1 before and after the collision, respectively, $\mathbf{v}_2$ and $\mathbf{v}_4$ are those of the particle of m_2, and all the velocities are referred to the local-ether frame.

Then, for an arbitrary velocity $\mathbf{v}_0$, one immediately has

$$m_1(\mathbf{v}_1 - \mathbf{v}_0) + m_2(\mathbf{v}_2 - \mathbf{v}_0) = m_1(\mathbf{v}_3 - \mathbf{v}_0) + m_2(\mathbf{v}_4 - \mathbf{v}_0). \tag{17}$$

This relation can be interpreted in the way that based on Galilean transformations the conventional momentum conserves as observed in a laboratory frame moving at the velocity $\mathbf{v}_0$ with respect to the local-ether frame. Further, by a direct expansion and by a use of (15), one has

$$m_1(\mathbf{v}_1 - \mathbf{v}_0)^2 + m_2(\mathbf{v}_2 - \mathbf{v}_0)^2 = m_1(\mathbf{v}_3 - \mathbf{v}_0)^2 + m_2(\mathbf{v}_4 - \mathbf{v}_0)^2. \tag{18}$$

Therefore, it is seen that the conventional momentum and kinetic energy conserve in any laboratory frame, in spite of the situation that their values of respective particles are different in different frames. That is, the conservation laws of conventional momentum and

kinetic energy are invariant under Galilean transformations and have no preferred frame, as expected in classical mechanics. Thus the local-ether model leads to consequences in accord with the conservation laws in classical mechanics and with Galilean relativity.

For the case with $m_2 \gg m_1$ and $\mathbf{v}_2 = \mathbf{v}_0$, it is seen that $\mathbf{v}_4 \simeq \mathbf{v}_0$ and then $(\mathbf{v}_3 - \mathbf{v}_0)^2 = (\mathbf{v}_1 - \mathbf{v}_0)^2$. Thus the particle speed, the magnitude of momentum, and the kinetic energy of a particle remain substantially unchanged before and after its collision with a rigid plane, when observed in the laboratory frame with respect to which the reflecting plane is stationary. Meanwhile, these two particle speeds in general are different when referred to the local-ether frame. Thus the wavelength of the reflected matter wave is different from that of the incident one. Since the difference between $\mathbf{v}_3$ and $\mathbf{v}_1$ is parallel to the normal $\hat{n}$ of the reflecting plane, it can be shown that

$$\mathbf{v}_3 = \mathbf{v}_1 - 2(\mathbf{v}_1 - \mathbf{v}_0) \cdot \hat{n}\hat{n}. \tag{19}$$

Moreover, one has $\hat{n} \cdot (\mathbf{v}_3 - \mathbf{v}_0) = -\hat{n} \cdot (\mathbf{v}_1 - \mathbf{v}_0)$. When the reflecting point is fixed on a rigid plane, the directions of the beams formed by the incident and the reflected particles are determined by the particle velocities with respect to the plane. Then the preceding relations lead to that the angle of reflection of a particle beam from the plane is equal to the angle of incidence, as observed in the laboratory frame. This consequence is simply in accord with the famous Snell's law of reflection.

5. Reexamination of Matter-Wave Interference Experiments

In this section, we discuss the matter-wave interferometry, where two matter waves are coherently split from a particle beam, propagate respectively along two separate paths, and then are combined to cause an interference depending on the phase difference. It is supposed that the phase variation ϕ of a matter wave over a differential path of directed length $d\mathbf{l}$ along the particle beam is given by

$$\phi = \mathbf{k} \cdot d\mathbf{l}, \tag{20}$$

where the particle velocity $\mathbf{v}$ associated with the propagation vector $\mathbf{k}$ $(= m\mathbf{v}/\hbar)$ is referred specifically to the local-ether frame, while the directed length $d\mathbf{l}$ is invariant in different reference frames. This expression is identical to the one adopted in [15, 16], except the reference frame of the particle velocity $\mathbf{v}$. Thereby, we reexamine the matter-wave interference experiments reported in the literature demonstrating the Bragg reflection, the double-slit diffraction, and the Sagnac effect, particularly the reference frame of particle velocity and the effects of earth's motions.

5.1. Bragg reflection and Young's slit diffraction of matter wave

The Bragg reflection from a crystal is due to the constructive interference among various reflected waves from successive lattice planes in parallel. Analogous to the Bragg reflection of x-ray from a crystal, the wavelength and the propagation constant of a matter wave can be determined from the scattering from the surface of lattice planes of a known spacing by measuring the Bragg angle θ_B, the reflection angle corresponding to the constructive interference.

For a crystal of lattice spacing d, the path-length difference between two waves either incident upon or reflected from two consecutive lattice planes is $d \cos \theta'$, where θ' is the angle of incidence and reflection measured from the normal $\hat{n}$ of the reflecting lattice plane when observed in the laboratory frame in which the crystal is stationary. Then, according to the aforementioned Snell's law of reflection, the Bragg condition of constructive reflection is given by

$$m(v_p + \mathbf{v}_0 \cdot \mathbf{t})d \cos \theta' = n\pi\hbar, \tag{21}$$

where $\mathbf{v}_p$ $(= \mathbf{v} - \mathbf{v}_0)$ represents the particle velocity with respect to the laboratory frame which in turn moves at the velocity $\mathbf{v}_0$ with respect to the local-ether frame, $\mathbf{t}$ is one half of the sum of the unit vectors $\hat{v}_p$ representing the directions of the incident and the reflected beams, and n is a positive integer. It is noted that the direction of $\mathbf{t}$ is parallel to the lattice planes and its magnitude $t = \sin \theta'$. It is seen that the Bragg angle depends on the

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laboratory speed and the orientation of the lattice planes with respect to the laboratory velocity.

Consider the ordinary case where the particle speed v is much lower than the speed of light and hence the particle mass is substantially identical to its rest mass m_0. In the mean time, the particle speed is supposed to be high enough such that $v_0 \ll v_p \ll c$. Thus the Bragg condition reduces to

$$m_0 v_p d \cos \theta' = n\pi\hbar. \tag{22}$$

As the laboratory speed v_0 is omitted, this formula becomes invariant under Galilean transformations. The preceding formula is identical to the one given in [1], if the matter wavelength therein is understood to be associated with the particle speed referred to the laboratory frame.

The matter wavelength has been demonstrated in the Davisson-Germer experiment, where the scattering of accelerated electron beams from the surface of a metal crystal was measured. Similarly, the matter wavelength has also been demonstrated in the diffraction ring pattern of a high-speed electron beam passing through a thin foil of polycrystalline gold [1]. More direct evidence for matter wavelength can be provided by the diffraction from macroscopic objects, such as Young's double slit. It is well known in optics that the period in the interference fringe pattern is proportional to the wavelength and to the inverse of the slit separation. Single-, double-, or multiple-slit diffraction of matter wave has been demonstrated for electron [17], neutron [18], and atom [19].

In these various experiments demonstrating the matter wavelength, the reference frame of the particle velocity is not explicitly specified and is supposed to be a geostationary laboratory frame. In what follows, we discuss the issue of reference frame and the effects of earth's rotation. Consider the isotropic case where the speed of the particles with respect to the ground is independent of the beam direction, such as the root-mean-square speed in thermal equilibrium at a given temperature. However, it is expected that for a given ground speed, the earthbound particles moving eastward will have the highest speed with respect to an ECI frame and hence have the shortest wavelength, while those moving westward, the longest. Thus the matter wavelength is expected to possess an east-west directional anisotropy due to earth's rotation, aside from the speed-dependence in the mass. In order to acquire a matter wavelength of one angstrom (the order of lattice spacing), the speed for electrons is 7.4×10^6 m/sec. This speed is much higher than the linear speed due to earth's rotation. Obviously, it makes no substantial difference whether the speed is referred to an ECI frame or to the ground. Therefore, the local-ether model is actually in accord with the Davisson-Germer experiment and others dealing with electron wave, as examined within the present precision.

On the other hand, for heavier particles, such as neutron and atoms, the effects of earth's rotation would be more appreciable. For example, in order to acquire a matter wavelength of one angstrom, the speed for neutrons is 4×10^3 m/sec, which is higher than the linear speed due to earth's rotation merely by a factor of about 10. In the Bragg reflection by a geostationary crystal, the ground speed of the reflected particles is identical to that of the incident ones. However, the wavelength of the reflected matter wave is expected to be different from that of the incident one, except when the Bragg angle is very small as measured from the crystal plane. Moreover, the Bragg angle and the fringe period in Young's slit diffraction are expected to depend on the orientation of the experimental setup with respect to the ground, even when the ground speed of the incident particles is identical.

5.2. Predicted anisotropy in Bragg angle due to earth's rotation

We then proceed to examine the minute effect due to the laboratory velocity. Suppose the Bragg angle θ_B is measured from the lattice plane (represented by the vector $\mathbf{t}$) to the reflected beam. Then the deflection angle of the particle beam measured from incidence to reflection is $2\theta_B$. For a given incident beam with an incident angle θ', there are two values for the Bragg angle θ_B, one positive and one negative, which are associated with two orientations of the crystal symmetric with respect to the incident beam, where $\theta_B = \pm(\pi/2 - \theta')$ and

$0 < \theta' < \pi/2$. Consider the case where the direction of the laboratory velocity $\mathbf{v}_0$ lies in the plane of incidence. Suppose the incident beam makes an angle α as measured from the direction of $\mathbf{v}_0$. Thus the vector $\mathbf{t}$ makes an angle $\alpha + \theta_B$ from that direction. Then the Bragg condition is given by

$$m_0[v_p + v_0 \cos(\alpha + \theta_B) \cos \theta_B]d|\sin \theta_B| = n\pi\hbar. \tag{23}$$

It is seen that the laboratory velocity complicates the dependence on angle θ_B.

Consider the Bragg reflection of neutron wave from a pyrolytic graphite crystal briefly reported in [15], where the laboratory is geostationary and the incident neutron beam is directed due south ($\alpha = 90°$). Then the Bragg condition becomes

$$|\sin \theta_B| \{1 - (v_0/v_p)\sin \theta_B \cos \theta_B\} = n\pi\hbar/m_0 v_p d. \tag{24}$$

The lattice constant is reported to be $d = 6.708$ angstrom with $n = 4$; the neutron wavelength is estimated to be $\lambda = 1.134$ angstrom and hence it is understood that $v_p = 3490$ m/sec; and the laboratory is located at a latitude of 38.63° and hence $v_0 = 362$ m/sec [15]. Then, according to the preceding formula, the predicted Bragg angles are $\theta_a = 20.48°$ and $\theta_b = -19.11°$. A positive or negative Bragg angle corresponds to the situation that the beam is reflected toward west or east, respectively. It is noted that the beam reflected toward west deflects more in magnitude than the one toward east. The difference in magnitude between the two Bragg angles is associated with the speed ratio v_0/v_p and presents an anisotropy due to earth's rotation.

The constructive reflection at the Bragg angle results in a minimum transmission which in turn can be detected by a counter. The measured results of the crystal rotation angle for minimum transmission are $\theta_1 = 19.14°$ and $\theta_2 = 159.63°$ [15], where the orientation corresponding to the starting value of the crystal rotation angle is not specified explicitly. It is understood to correspond to the orientation at which the normal of the reflecting crystal surface is directed due west. Thus θ_1 and θ_3 ($= \theta_2 - 180° = -20.37°$) should be identical to θ_a and θ_b, respectively. It is seen that the two Bragg reflections are asymmetric, as predicted by the local-ether model. However, the agreement is not good quantitatively. The discrepancies in the Bragg angles might be due to that the incident beam is not exactly directed due south or that the crystal rotation angle should be understood otherwise. Anyway, the predicted directional anisotropy in the Bragg angle due to earth's rotation provides a means to test the local-ether wave equation.

5.3. Matter-wave Sagnac effect in rotating loop

By using the Bragg reflection from multiple crystals to form a closed path for the particle beam, the loop interferometry of matter wave can be achieved. We discuss the Sagnac effect of matter wave associated with the rotation of an interferometer which has a closed path formed by a series of reflecting planes of suitable orientations. Consider two coherently split particle beams form two paths L_1 and L_2 which in turn form a coplanar closed contour L of arbitrary shape. Suppose the loop L is rotating about an axis at an arbitrary location with an arbitrary directed rotation rate $\bar{\omega}_I$ with respect to the laboratory frame which in turn is rotating in an ECI frame. Thus the velocity $\mathbf{v}_l$ of the various reflecting planes forming the path is not uniform and is given by $\mathbf{v}_l = \mathbf{v}_0 + (\bar{\omega}_I + \bar{\omega}_E) \times (\mathbf{r} - \mathbf{r}_0)$ with respect to the local-ether frame, where $\mathbf{r}$ and $\mathbf{r}_0$ denote the position vectors of the planes and of a suitable reference point at which the laboratory velocity $\mathbf{v}_0$ is defined, respectively, and $\bar{\omega}_E$ is the directed rate of earth's rotation.

The direction of the particle beam around the loop is determined by the propagation vector $\mathbf{k}_p$ ($= m\mathbf{v}_p/\hbar$), where $\mathbf{v}_p$ ($= \mathbf{v} - \mathbf{v}_l$) is the particle velocity with respect to the associated reflecting plane. For the trivial case with $\omega_I = 0$, the path velocity $\mathbf{v}_l$ is substantially uniform among the planes, as ω_E is quite small. Thus the particle velocity $\mathbf{v}_p$, the propagation vector $\mathbf{k}_p$, and the particle beam traveling between two consecutive planes are all along the directed linear segment $\mathbf{l}$ joining the reflecting point on one of the two planes to that on

the other. Consequently, the term $\mathbf{k}_p \cdot \mathbf{l}$ incorporated in the phase variation $\mathbf{k} \cdot \mathbf{l}$ over the path segment $\mathbf{l}$ is then equal to the scalar product $k_p l$. Furthermore, the particle speed v_p and the propagation constant k_p are always fixed while the particle beam is traveling around the loop. On the other hand, for the case with $\omega_I \neq 0$, the path velocity $\mathbf{v}_l$ is varying among the planes. Thus the direction of the particle beam, as well as those of the particle velocity $\mathbf{v}_p$ and of the propagation vector $\mathbf{k}_p$, tends to deviate from the path segment $\mathbf{l}$. However, the effect of these deviations on the phase variation is small if the rotation rate ω_I times the segment length l is much smaller than the particle speed v_p. Actually, it is merely of the second order of the speed ratio $\omega_I l / v_p$, since the velocity difference between two consecutive planes under rotation is perpendicular to the segment $\mathbf{l}$. Similarly, the fractional variation in the magnitude k_p around the loop is also of this order.

Thereby, for a loop formed by closely spaced reflecting planes or undergoing a slow rotation such that the condition $\omega_I l \ll v_p$ is met for every segment, the phase difference between the coherent beams along the two paths can be given by the path integral

$$\triangle\phi = -\int_{L_1} \mathbf{k} \cdot \hat{l}_1 dl \;+\; \int_{L_2} \mathbf{k} \cdot \hat{l}_2 dl = \oint_L \mathbf{k} \cdot d\mathbf{l}, \tag{25}$$

where $d\mathbf{l} = -\hat{l}_1 dl$ and $\hat{l}_2 dl$ on paths L_1 and L_2, respectively. Further, as the differential phase variation $\mathbf{k}_p \cdot d\mathbf{l}$ can be given by the scalar product $k_p dl$ in conjunction with k_p being a constant value around the loop, the phase difference becomes

$$\triangle\phi = k_p(l_2 - l_1) + \frac{m_0}{\hbar} \oint_L \mathbf{v}_l \cdot d\mathbf{l}, \tag{26}$$

where l_1 and l_2 are the lengths of paths L_1 and L_2, respectively. In deriving the preceding formula, the ordinary condition $v \ll c$ is assumed such that the speed-dependent mass m is substantially identical to the rest mass m_0.

For the case where the path is suitably structured such that $l_1 = l_2$, the phase difference then takes the form of

$$\triangle\phi = \frac{2\omega_0}{c^2}(\bar{\omega}_I + \bar{\omega}_E) \cdot \mathbf{S}, \tag{27}$$

where $\mathbf{S}$ $(= \frac{1}{2} \oint_L \mathbf{r} \times d\mathbf{l})$ is the directed area enclosed by the loop L and we have made use of a vector identity, the relation $\hbar\omega_0 = m_0 c^2$, and of the result that the integration of a constant vector $\mathbf{v}_0$ or $\mathbf{r}_0$ around an arbitrary loop is zero. It is seen that earth's rotation as well as the rotation of the loop with respect to the ground contributes to the phase difference, as the path velocity $\mathbf{v}_l$ is referred to an ECI frame for terrestrial experiments and hence incorporates the linear velocity due to earth's rotation. The phase-difference formula (27) with either $\bar{\omega}_I$ or $\bar{\omega}_E$ looks like the matter-wave Sagnac effect derived in [15, 20-22] from various different approaches. The matter-wave Sagnac effect associated with the rotation-induced quantum interference has been demonstrated with a particle beam of electron [22], neutron [15, 16], or of atom [23, 24]. In these experiments, the loop can be rotating on a turntable [22, 23] or simply be geostationary with variable orientation [15, 16]. For a geostationary loop with a specific structure, the phase difference due to earth's rotation together with that due to earth's gravity has also been derived from the local-ether model [25], which agrees with the experimental results with neutron beam reported in [15, 16].

It is noted that the phase difference (27) is identical to the one for optical wave when the natural frequency ω_0 is replaced with the angular frequency of an optical wave, aside from a factor of 2 due to a difference in the path structure [3]. As the natural frequency of matter wave is much higher than an optical frequency, the loop area required for the matter-wave interference can be much smaller. By using the geostationary loop interferometer with neutron wave, the phase difference (27) due to earth's rotation alone has been demonstrated with a high precision. It is noticed that the loop area employed for the detection of earth's rotation with neutron wave is about 10^{-3} m^2 in [15, 16], which is much smaller than 2×10^5

m^2 in the Michelson-Gale experiment for the same purpose with optical wave [26]. By using cesium atomic beam, earth's rotation has also been detected with the Sagnac loop interferometry [24].

Meanwhile, in as early as 1904 Michelson supposed that the Sagnac effect due to the orbital motion of the Earth around the Sun might be detectable [26], although the angular speed of the orbital motion is about 1/365 times that of the rotation. If this orbital effect does exist, it could be demonstrated with matter wave of neutron or atom by constructing a path of area of 0.2 m^2 or smaller. However, the orbital effect is never observed in terrestrial interferometers either with electromagnetic or matter wave, to our knowledge. In some earthbound experiments with electromagnetic wave, this effect has been assumed to be null by resorting to the principle of local Lorentz invariance (see [3] for a discussion). Anyway, according to the local-ether propagation model, the particle velocity which determines the propagation vector **k** of matter wave is referred uniquely to an ECI frame, rather than to a heliocentric inertial, the ECEF (earth-centered earth-fixed), or any other frame. Thus earthbound experiments can depend on earth's rotation but are entirely independent of earth's orbital motion around the Sun or whatever. Thereby, the local-ether model unambiguously predicts a discrepancy between the effects of the rotational and the orbital motions of the Earth in the Sagnac loop interferometry as well as in many other earthbound experiments discussed in [3, 4], which provides a means to test its validity. Moreover, the phase-difference formula (27) is not expected to hold for an interferometer with a high rotation rate, especially when the particle speed v_p is low. This restriction provides another means to test the local-ether wave equation.

6. Conclusion

The local-ether wave equation for a free particle looks like the free-space Klein-Gordan equation, except the reference frame of the time derivative. By virtue of the natural frequency, the wave equation leads to a dispersion relation for a harmonic-like wave packet. From the dispersion relation, the angular frequency and wavelength of matter wave and the speed-dependent mass of particle are derived, which look like the postulates of de Broglie and the Lorentz mass-variation law, respectively, except the reference frame of the particle velocity. For a harmonic matter wave, the angular frequency and the propagation vector are associated with the energy and the momentum of the particle, respectively, as the energy and the momentum are given by the expectation values of the temporal and the spatial derivatives evaluated in the wavefunction, respectively. As the phase variation of a particle beam is given by the propagation vector and the path length along the beam, a shift in these quantities leads to a phase difference between two coherent beams. From the difference in the path length, the Bragg condition of constructive reflection from a crystal is derived. Moreover, from the shift in the propagation vector due to earth's rotation, the rotation of the interferometer, or to the gravity, the phase differences in the loop interferometry are derived.

As the velocity of earthbound particles is referred uniquely to an ECI frame, rather than a laboratory frame or any other, it is predicted that earth's rotation leads to an east-west directional anisotropy both in mass and wavelength. Moreover, the particle velocity involved in the matter-wave interferometry incorporates the laboratory velocity with respect to an ECI frame for terrestrial experiments. For electrons, the mass is light and the speed is normally high. The anisotropy due to earth's rotation or the effect due to the laboratory velocity is then negligibly small. Thereby, the local-ether model for electron wave is actually in accord with the Davisson-Germer experiment and Young's slit diffraction as examined within the present precision. Further, the effect of the laboratory velocity can cancel out in a loop interferometer. Consequently, in spite of the restriction on the reference frame of particle velocity, the derived interference formulas for the Bragg reflection or for the loop interferometry can be independent of the laboratory velocity and then comply with Galilean relativity.

On the other hand, for heavier particles of neutron or atom, the effects of earth's rotation

on matter wavelength are expected to be more appreciable. It is predicted that the Bragg angle and the fringe period in Young's slit diffraction depend on the orientation of the experimental setup with respect to the ground. For a given incident neutron beam, it is analyzed that the Bragg angle can deviate by an amount of about one degree between two symmetric orientations of the crystal. The predicted directional anisotropy in the Bragg angle and in the fringe period, the slow-rotation restriction on the famous formula of matter-wave Sagnac effect, and the discrepancy between the effects of earth's rotational and orbital motions provide different approaches to test the local-ether wave equation.

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