Resonant Absorption between Moving Atoms due to Doppler Frequency Shift and Quantum Energy Variation

Ching-Chuan Su
Department of Electrical Engineering
National Tsinghua University
Hsinchu, Taiwan

Abstract – By taking both the Doppler frequency shift for electromagnetic wave and the quantum energy variation of matter wave into consideration, a resonant-absorption condition based on the local-ether wave equation is presented to account for a variety of phenomena consistently, including the Ives-Stilwell experiment, the output frequency from ammonia masers, and the Mössbauer rotor experiment. It is found that in the resonant-absorption condition, the major term associated with the laboratory velocity is a dot-product term between this velocity and that of the emitting or absorbing atom. This term appears both in the Doppler frequency shift and the transition frequency variation and then cancels out. Thereby, the experimental results can be independent of the laboratory velocity and hence comply with Galilean relativity, despite the restriction that the involved velocities are referred specifically to the local-ether frame. However, by examining the resonant-absorption condition in the Mössbauer rotor experiment to a higher order, it is found that Galilean relativity breaks down.

1. Introduction

It is well known that the Doppler effect due to the relative motion between a transmitter and a receiver causes a shift in the received frequency of electromagnetic or acoustic wave. The Doppler effect has been applied by Ives and Stilwell to deal with the frequency shift in the light emitted from a fast-moving hydrogen atomic beam [1]. However, it was found that the observed frequency shift agrees with the Doppler effect only to the first order of the atom speed normalized to the speed of light c. In order to account for the observed shift correctly to the second order, a hypothesis of Larmor and Lorentz was adopted, which states that the frequency of a wave radiated from a moving source of speed v is altered by a factor of $\sqrt{1-v^2/c^2}$ [1]. It is noted that this speed-dependent factor is identical to the one in the famous Lorentz mass-variation law. Presently, an almost unanimously accepted explanation of this additional frequency shift is provided by Einstein's special relativity, whereby a second-order Doppler effect has been derived based on the Lorentz transformation of space and time [2]. Thus it seems to imply that the frequency shift to the second order between a transmitter and a receiver in relative motion is purely a kinematical property. And, for the case of resonant absorption between moving atoms or ions, it seems to assume tacitly that the transition frequencies of the emitting or the absorbing atoms in motion remain unchanged, in spite of the well-known fact that the electronic quantum states in atoms depend on the mass of electron, which in turn depends on the particle speed. Furthermore, in the common understanding the speed v which determines the second-order Doppler frequency shift by the mass-variation factor is the relative speed between the transmitter and the receiver. However, in the Hafele-Keating experiment with circumnavigating atomic clocks and in the GPS (global positioning system) with atomic clocks onboard the orbiting satellites, it has been demonstrated with a high precision that the speed which determines the transition frequency and clock rate by the mass-variation factor is referred uniquely to an ECI (earth-centered inertial) frame [3]. Thus there seems to exist a discrepancy in the reference frame for the speed in the mass-variation factor.

Recently, we have presented the local-ether model of propagation of electromagnetic wave [4]. It is supposed that electromagnetic wave propagates via a medium like the ether. However, the ether is not universal. Specifically, it is supposed that in the region under sufficient influence of the gravity due to the Earth, the Sun, or another celestial body, there forms a local ether which as well as the associated gravitational potential moves with the respective body. Within each local ether, it is proposed that as in the classical propagation model, electromagnetic wave propagates at the speed of light c with respect to the associated local ether, independent of the motions of source and receiver. Thereby, for earthbound or interplanetary waves, the propagation is referred specifically to a geocentric or a heliocentric inertial frame, respectively. This local-ether model has been adopted to account for the effects of earth's motions in a wide variety of propagation phenomena, particularly the Sagnac correction in GPS, the time comparison via intercontinental microwave link, and the echo time in interplanetary radar. As examined within the present accuracy, the local-ether model is still in accord with the Michelson-Morley experiment which is known to make the classical ether notion obsolete. Furthermore, by modifying the speed of light in a gravitational potential, this simple propagation model leads to the deflection of light by the Sun and the increment in the interplanetary radar echo time which are important phenomena supporting the general theory of relativity. Moreover, based on this new classical model, the first-order Doppler frequency shift has been derived to account for the anisotropy in antenna temperature of CMBR (cosmic microwave background radiation), the shift in the spectrum of light radiated from a moving star, and the seasonal variation of eclipse intervals in Roemer's observations of Jupiter's moons [4].

Further, the matter wave associated with a particle has been supposed to follow the local-ether model and be governed by a wave equation incorporating a natural frequency and the electric scalar potential [3]. From the local-ether wave equation, a first-order time evolution equation similar to Schrödinger's equation is derived. From the electrostatic force derived from this evolution equation, it has been found that the rest mass of a particle is just the natural frequency, aside from a scaling factor. That is, the inertial mass of a particle originates from the natural frequency and hence from the temporal variation of the associated matter wave. Furthermore, due to the dispersion of matter wave, it has been derived that the mass of the particle and the angular frequency of the matter wave increase with the particle speed by the famous Lorentz mass-variation factor, except that the speed is referred specifically to the associated local-ether frame. A feature different from Schrödinger's equation is that the time derivative in the local-ether evolution equation incorporates an extra multiplying term of the ratio of the speed-dependent angular frequency to the natural frequency. As a consequence, it has been found that the energies of quantum states of the matter wave bounded in an atom decrease with the inverse of the speed-dependent mass [3]. Thus the transition frequency and the atomic clock rate decrease with the atom speed by the mass-variation factor. Thereby, the frequency variation associated with the speed-dependent mass-variation factor has been derived as an intrinsic quantum property of the matter wave bounded in atom. Similarly, the gravitational redshift has been derived as an effect of the gravitational potential on the quantum energy. Anyway, the dependence of quantum energy on speed can be expected at least for hydrogen-like atoms. Since it is well known in standard textbooks on quantum mechanics that the electronic quantum states in such an atom depend on the mass of electron, which in turn has been known to depend on speed. However, according to the local-ether model, the atom speed that determines the mass variation is referred uniquely to an ECI (earth-centered inertial) frame for earthbound atoms. Thus the quantum energy and hence the atomic clock rate tend to depend on earth's rotation, but is entirely independent of earth's orbital motion around the Sun or others. This consequence has been used to account for the east-west directional anisotropy in the Hafele-Keating experiment, the synchronism among GPS atomic clocks, and for the spatial isotropy in the Hughes-Drever experiment [3].

In this investigation, based on the local-ether propagation model, a higher-order Doppler frequency shift for electromagnetic wave is derived. Further, by taking the speed-dependent variation of quantum energy of matter wave into account, we present a resonant-absorption condition between moving emitting and absorbing atoms. Then this frequency relation is applied to deal with various experiments consistently, including the Ives-Stilwell experiment, the output frequency from ammonia masers, and the Mössbauer rotor experiment. Moreover, we examine the spatial isotropy, Galilean relativity, and their breakdowns in these experiments.

2. Higher-Order Doppler Frequency Shift

In this section, based on the local-ether model of wave propagation, the Doppler frequency shift is given to higher orders. Consider the case where both the source and the receiver are located within the same local ether and are moving at velocities $\mathbf{v}_s$ and $\mathbf{v}_e$ with respect to this local-ether frame, respectively. According to the classical propagation model, the propagation time τ is the propagation range R divided by the isotropic speed c as $\tau = R/c$. It is important to note that the propagation range is the distance from the position of the source at the instant of wave emission to the one of the receiver at the instant of reception, as viewed in the local-ether frame. Thus the propagation range and hence the propagation time depend on the velocity $\mathbf{v}_e$ and the acceleration $\mathbf{a}_e$ of the receiver with respect to the local-ether frame. Quantitatively, when τ is short, the propagation range is given implicitly by

$$R = \left| \mathbf{R}_t + \mathbf{v}_e \frac{R}{c} + \mathbf{a}_e \frac{R^2}{2c^2} \right|, \tag{1}$$

where $\mathbf{R}_t$ is the directed propagation-path length from the source to the receiver both at the instant of emission. It can be shown that to the second order of normalized speed, the propagation range can be given explicitly in terms of the path length R_t by [4]

$$R = R_t \left\{ 1 + \frac{1}{c} u_e + \frac{1}{2c^2} \left(u_e^2 + v_e^2 + \mathbf{a}_e \cdot \mathbf{R}_t \right) \right\}, \tag{2}$$

where radial speed u $(= \mathbf{v} \cdot \hat{R}_t)$ is the component of velocity $\mathbf{v}$ along $\mathbf{R}_t$ and the unit vector $\hat{R}_t = \mathbf{R}_t / R_t$.

Consider the Doppler effect for the case where the source is emitting wave periodically and is moving with respect to the receiver. Due to the motions of the transmitter and receiver, the rate of reception tends to be different from the one of emission. As discussed in [4], the received time difference $\triangle t$ between two signals transmitted with a differential time difference $\triangle t'$ can be given in terms of the difference in the propagation range by

$$\triangle t = \triangle t' + \frac{R(t' + \triangle t')}{c} - \frac{R(t')}{c}, \tag{3}$$

where $R(t)$ denotes the propagation range for the wave emitted at an arbitrary instant t. Then the received frequency f_r and the transmitted frequency f_t are related by

$$\frac{f_t}{f_r} = \frac{\triangle t}{\triangle t'} = 1 + \frac{dR}{cdt}, \tag{4}$$

where the time derivative of the propagation range is evaluated at the instant of wave emission.

The second-order Doppler effect can be given if the first-order formula of the propagation range is used. By so doing, the Doppler frequency relation becomes

$$\frac{f_t}{f_r} = 1 + \frac{dR_t}{cdt} + \frac{d}{c^2 dt} (\mathbf{v}_e \cdot \mathbf{R}_t). \tag{5}$$

Due to the relative motion between source and receiver, $d\mathbf{R}_t/dt = \mathbf{v}_{es}$ and hence $dR_t/dt = u_{es}$, where $\mathbf{v}_{es}$ $(= \mathbf{v}_e - \mathbf{v}_s)$ is the Newtonian relative velocity between receiver and source at the instant of emission and u_{es} $(= u_e - u_s)$ is the radial speed of the receiver with respect to the source. Thus, as given in [4], the second-order Doppler frequency relation becomes

$$\frac{f_t}{f_r} = 1 + \frac{u_{es}}{c} + \frac{\mathbf{v}_e \cdot \mathbf{v}_{es}}{c^2} + \frac{\mathbf{a}_e \cdot \mathbf{R}_t}{c^2}. \tag{6}$$

It is noted that the transverse components of velocities are also involved in the second-order Doppler shift. To the second order of normalized speed, the inverse frequency ratio reads

$$\frac{f_r}{f_t} = 1 - \frac{u_{es}}{c} - \frac{\mathbf{v}_e \cdot \mathbf{v}_{es}}{c^2} - \frac{\mathbf{a}_e \cdot \mathbf{R}_t}{c^2} + \frac{u_{es}^2}{c^2}. \tag{7}$$

When the third-order Doppler effect needs to be considered, the second-order formula of the propagation range should be used, as discussed later.

3. Quantum Energy Variation and Resonant-Absorption Condition

Based on the local-ether wave equation, it has been found that the mass of a particle increases with its speed by the famous Lorentz mass-variation factor, except that the speed is referred specifically to an ECI frame for earthbound particles [3]. From this wave equation a first-order time evolution equation is derived, which is similar to Schrödinger's equation except that the time derivative is referred specifically to the local-ether frame and incorporates an extra multiplying term of the mass-variation factor. Thereby, a quantum-mechanical approach has been presented to show that the energies of quantum states of the matter wave bounded in an atom or a molecule decrease with the inverse of the speed-dependent mass. It is known that the frequency of light emitted from or absorbed by an atom is equal to the transition frequency, which in turn corresponds to the difference in energy between two involved quantum states. Thus the state transition frequency f of a moving atom decreases with its speed by the speed-dependent mass-variation factor as [3]

$$f = f_0\sqrt{1 - v^2/c^2}, \tag{8}$$

where v is the atom speed referred to the local-ether frame which is an ECI frame for earthbound atoms, f_0 is the rest transition frequency of an identical atom stationary in the local-ether frame, and the frequency f is observed in the atom frame (with respect to which the atom is stationary) such that no Doppler shift is involved.

The atomic clock rate depends on the transition frequency of the associated atom and hence decreases with the atom speed by the mass-variation factor. The preceding speed-dependent frequency-variation formula has been adopted in [3] to account for the east-west directional anisotropy in the clock rate demonstrated in the Hafele-Keating experiment with cesium atomic clocks onboard an aircraft undergoing circumnavigation [5]. In this experiment the observed clock rate is determined from the number of the atomic clock ticks which in turn are detected and counted by a device which is comoving with the clock without relative motion. Thus the clock rate is associated with the quantum energy variation, but has nothing to do with the Doppler frequency shift. Moreover, it has been used to account for the synchronism and the clock-rate adjustment in GPS atomic clocks onboard earth's satellites in circular orbits.

A speed-dependent frequency of the form of (8) (but of different physical origin and reference frame of speed) was first introduced by Fitzgerald, Lorentz, and Larmor before the advent of the special relativity [6] and was later derived by assuming the length contraction [7] or the time dilation [2]. In the local-ether model, the frequency-variation formula due to the quantum effect is not expected to hold in all energy states in all atoms and other forms of the speed-dependence are not precluded. This is in view of the complication that the interactions which affect energy states are versatile, such as electronic or nuclear, electric or magnetic, spin or orbital, and intrinsic or external. In [3], the frequency-variation formula is verified only for electronic states due to the electric scalar potential in an atom or a molecule. Nevertheless, it is assumed that this formula holds in the experiments examined in this investigation.

Then we proceed to consider the case where the source atom s and the receiver atom e are located within the same local ether but are moving respectively at velocities $\mathbf{v}_s$ and $\mathbf{v}_e$ with respect to the local-ether frame. Due to the motions of the source and receiver, the Doppler effect also causes a frequency shift. Suppose f_{0s} and f_{0e} are the rest transition frequencies of atoms s and e, respectively. The transmitted frequency f_t of the wave emitted from the moving atom s is related to the rest transition frequency f_{0s} as

$$f_t = f_{0s}\sqrt{1 - v_s^2/c^2}. \tag{9}$$

On the other hand, in order for the wave being resonantly absorbed by the moving atom e, the rest transition frequency f_{0e} should be related to the received frequency f_r as

$$f_r = f_{0e}\sqrt{1 - v_e^2/c^2}. \tag{10}$$

The received frequency f_r and the transmitted frequency f_t in turn are related to each other by the Doppler effect as shown in (6) or (7).

Thus, by taking both the Doppler frequency shift for electromagnetic wave and the quantum energy variation of matter wave into account, we arrive at the **resonant-absorption condition** in terms of the rest transition frequencies f_{0s} and f_{0e}, which to the second order of normalized speed is given by

$$f_{0e}\sqrt{1 - v_e^2/c^2} = f_{0s}\sqrt{1 - v_s^2/c^2}\left\{1 - u_{es}/c - (\mathbf{v}_e \cdot \mathbf{v}_{es} + \mathbf{a}_e \cdot \mathbf{R}_t - u_{es}^2)/c^2\right\}. \tag{11}$$

For the case where the receiver is comoving with the source ($\mathbf{v}_e = \mathbf{v}_s$) without acceleration, the resonant-absorption condition becomes trivially as $f_{0e} = f_{0s}$. This resonant-absorption condition made its debut in [8].

When the resonant-absorption condition is not met, the off-resonance absorption tends to be much weaker. The deviation in frequency from the resonant-absorption condition is given by the difference between the received frequency and the transition frequencies of the absorbing atoms in motion. That is,

$$\delta f = f_r - f_{0e}\sqrt{1 - v_e^2/c^2}. \tag{12}$$

By using (9), the frequency deviation can be written as

$$\delta f = \triangle f_D - \triangle f_Q, \tag{13}$$

where $\triangle f_D$ and $\triangle f_Q$ denote the frequency shifts due to the Doppler and the quantum effects and are given by

$$\triangle f_D = f_r - f_t \tag{14}$$

and

$$\triangle f_Q = f_{0e}\sqrt{1 - v_e^2/c^2} - f_{0s}\sqrt{1 - v_s^2/c^2}, \tag{15}$$

respectively. In words, $\triangle f_Q$ is the difference in the speed-dependent transition frequency between the receiver and source atoms in motion. From (4) it is seen that the fractional Doppler frequency shift is given by

$$\frac{\triangle f_D}{f_r} = -\frac{dR}{cdt}. \tag{16}$$

The resonant-absorption condition corresponds to the deviation $\delta f = 0$, which means that the frequency shift due to the Doppler effect for electromagnetic wave propagating in free space exactly cancels the one due to the quantum effect of matter wave bounded in atom.

4. Reexamination of Resonant-Absorption Experiments

Based on the local-ether model, we reexamine the resonant-absorption experiments reported in the literature. Thereby, we present reinterpretations of the Ives-Stilwell experiment, the output frequency from ammonia masers, and of the Mössbauer rotor experiment and explore the associated spatial isotropy, Galilean relativity, and their breakdowns. The first experiment is associated with a radial relative motion ($u_{es} \simeq \pm v_{es}$) and the last two are with a transverse relative motion ($u_{es} = 0$).

4.1. Ives-Stilwell experiment

Consider the case where the source is moving at a velocity $\mathbf{v}_s$ and the receiver is stationary, both with respect to the local-ether frame. Thus $a_e = 0$, $v_e = 0$, and $u_{es} = -u_s$. Then the second-order resonant-absorption condition becomes a simpler form of

$$f_{0e} = f_{0s} \frac{\sqrt{1 - v_s^2/c^2}}{1 - u_s/c}. \tag{17}$$

For this case $f_r = f_{0e}$ and thus this formula is identical to that derived in different ways based on the transformation of four-vectors [9] or on the time dilation [2], except the difference in reference frame of velocity $\mathbf{v}_s$. According to the local-ether model, the term $(1 - u_s/c)$ in the preceding formula is due to the Doppler frequency shift to the second order, while the term $\sqrt{1 - v_s^2/c^2}$ is due to the quantum energy variation.

A similar frequency shift has been demonstrated in the Ives-Stilwell experiment, where light is emitted from fast-moving hydrogen atoms in an excited state and is absorbed by a spectrograph used to measure the received frequency [1, 10]. The radiation from the moving atoms is reflected from two mirrors and then the two reflected light beams are guided to a photographic plate where the frequencies are recorded for measurement and comparison. The two mirrors are arranged in such a way that the atoms are moving toward one of them and away from the other [10]. Due to the Doppler effect with the relative motions between the emitting atoms and the mirrors, the light beams reflected from the mirrors tend to shift in frequency. However, by virtue of no relative motions among the mirrors, the spectrograph, and the other components of experimental setup, no further Doppler effect is introduced after the reflection.

Note that even for a geostationary experimental setup, the receiver is not stationary with respect to the associated local-ether frame due to earth's rotation. Consider the laboratory frame in which the setup and the receiver are stationary. Suppose that the emitting atoms are moving at a velocity $\mathbf{v}_t$ with respect to the laboratory frame, which in turn moves at a velocity $\mathbf{v}_0$ with respect to an ECI frame. Thus $\mathbf{v}_e = \mathbf{v}_0$, $\mathbf{v}_s = \mathbf{v}_t + \mathbf{v}_0$, and $\mathbf{v}_{es} = -\mathbf{v}_t$. In the experiment the atoms move nearly in the radial direction and the directions of the two propagation paths are antiparallel. Thus $u_{es} = \mp v_t \cos\theta$, where θ denotes the small angle of $\mathbf{v}_t$ from $\hat{R}_t$, $\pm\hat{R}_t$ represents the direction from the emitting atoms to the reflecting point on either mirror, and the upper or the lower sign applies for the light beam reflected from the mirror which the atoms approach or recede from, respectively. Then the resonant-absorption condition (11) leads to that the received frequencies for the two reflected light beams are

$$f_{r\pm} = f_{0s}\sqrt{1 - (\mathbf{v}_t + \mathbf{v}_0)^2/c^2}\left\{1 \pm v_t \cos\theta/c + (\mathbf{v}_0 \cdot \mathbf{v}_t + v_t^2 \cos^2\theta)/c^2\right\}. \tag{18}$$

It is noted that the first-order term is independent of the laboratory velocity $\mathbf{v}_0$. If the velocity $\mathbf{v}_0$ is neglected, the preceding formula reduces to (17).

To the second order of normalized speed, the received frequencies become

$$f_{r\pm} = f_{0s}\left\{1 \mp v_t \cos\theta/c + (v_t^2 + v_0^2)/2c^2\right\}^{-1}. \tag{19}$$

Both the Doppler and the quantum effects contribute to the second-order terms. It is noted that the dot-product term $\mathbf{v}_0 \cdot \mathbf{v}_t$ appears both in the Doppler and the quantum effects and then cancels out. In the Ives-Stilwell experiment the speed v_t of hydrogen atoms is of the order of 10^6 m/sec, which is achieved by electrostatically accelerating hydrogen ions with a voltage of tens of kV. Thus the atom speed v_t is much higher than the laboratory speed v_0 which is due to earth's rotation and, perhaps, to the motion of a vehicle carrying the experimental setup. Thus the preceding frequency formula can be substantially independent of the laboratory velocity $\mathbf{v}_0$.

The first- and the second-order fractional frequency shifts are associated with the difference between and with the sum of the two received frequencies, respectively. Thus, in terms of wavelengths, the normalized speed and its square can be given by

$$\frac{v_t}{c} = \frac{1}{2\cos\theta}\frac{\lambda_R - \lambda_B}{\lambda_0} \tag{20}$$

and

$$\frac{v_t^2}{c^2} = \left(\frac{\lambda_R + \lambda_B}{\lambda_0} - 2 - \frac{v_0^2}{c^2}\right), \tag{21}$$

where $\lambda_B = c/f_{r+}$, $\lambda_R = c/f_{r-}$, and $\lambda_0 = c/f_{0s}$. By measuring λ_R and λ_B, the first- and the second-order fractional shifts can be calculated and compared, with a knowledge of θ, λ_0, and v_0. This provides a crucial means to test the minor second-order frequency shift. It can be expected that the deviation of the sum $\lambda_R + \lambda_B$ from $2\lambda_0$ is quite small. By using a high-precision spectrograph, it has been measured that $\lambda_R = 6618.808$ (in angstrom) and $\lambda_B = 6507.253$ [10]. Meanwhile, it is known that $\lambda_0 = 6562.793$ and $\theta = 0.075$ (in rad). Then formulas (20) and (21) lead to $v_t^2/c^2 = 7.26 \times 10^{-5}$ and 7.24×10^{-5}, respectively, where the term v_0^2/c^2 is as small as 10^{-12} and hence is omitted. The agreement between the two data is quite good, since the discrepancy is as small as the round-off in the calculation. According to the local-ether model, the wavelength λ_0 corresponds to a transition frequency of a hydrogen atom stationary with respect to an ECI frame. This seems to be in accord with the description that λ_0 is the wavelength of the H_α line of the Balmer series observed in a reference frame at rest with respect to the radiating atom [10]. However, the value of λ_0 could be actually measured from geostationary atoms in a geostationary laboratory. Even so, the fractional discrepancy in λ_0 due to the difference in reference frame is also of the order of v_0^2/c^2. Thus the local-ether model is in accord with the self-consistency between the first- and the second-order frequency shifts demonstrated in the Ives-Stilwell experiment.

4.2. Output frequency from ammonia masers

Next, we consider the case where the source is moving transverse to the propagation path. In the ammonia maser, the molecular beam is injected into a resonant cavity and emits microwave due to quantum state transition [11, 2]. The output of the microwave comes through a waveguide coupled to the cavity via a small aperture. Suppose that the cavity and the receiver are stationary in the laboratory frame of velocity $\mathbf{v}_0$ and that the emitting ammonia molecular beam moves at a velocity $\mathbf{v}_t$ with respect to the cavity. Moreover, suppose that the output waveguide is thin and long and is positioned with its longitudinal axis being perpendicular to the direction of molecular beam represented by $\mathbf{v}_t$. Thus the direction of the propagation path of the microwave is virtually parallel to the longitudinal axis of the output waveguide and hence is transverse to the molecule velocity, that is, $\mathbf{v}_t \cdot \hat{R}_t = 0$.

Thus $\mathbf{v}_e = \mathbf{v}_0$, $\mathbf{a}_e = 0$, $\mathbf{v}_s = \mathbf{v}_t + \mathbf{v}_0$, $\mathbf{v}_{es} = -\mathbf{v}_t$, and $u_{es} = 0$. Then the resonant-absorption condition (11) leads to the received frequency

$$f_r = f_{0s}\sqrt{1 - (\mathbf{v}_t + \mathbf{v}_0)^2/c^2}\left\{1 + \mathbf{v}_0 \cdot \mathbf{v}_t/c^2\right\}. \tag{22}$$

The Doppler effect is presented by the dot-product term $\mathbf{v}_0 \cdot \mathbf{v}_t$. Thus, by taking both the Doppler and the quantum effects into account, the resonant-absorption condition leads to that the received frequency at the output waveguide of the maser is given by

$$\frac{f_r}{f_{0s}} = 1 - \frac{v_t^2 + v_0^2}{2c^2}. \tag{23}$$

It is noted again that the dot-product term $\mathbf{v}_0 \cdot \mathbf{v}_t$ appears both in the Doppler and the quantum effects and then cancels out. Consequently, the output frequency from the maser is independent of the directions of velocities $\mathbf{v}_t$ and $\mathbf{v}_0$. Moreover, the resonant-absorption condition leads to

$$\frac{f_{0e}}{f_{0s}} = 1 - \frac{v_t^2}{2c^2}. \tag{24}$$

It is seen that this frequency relation is even independent of the laboratory velocity $\mathbf{v}_0$.

It is noticed that a term quite similar to the second-order Doppler effect in (22) has been derived alternatively from a classical approach, aside from the reference frame of the laboratory velocity [11, 2]. Further, a similar cancellation of the terms associated with $\mathbf{v}_0$ has been accounted for by an alternative approach, where a speed-dependent frequency based on the time dilation is used [12]. Anyway, it has been demonstrated experimentally that the beat frequency between two masers with molecular beams moving in opposite directions is substantially zero and is almost independent of earth's motions [11]. Thus the local-ether model is in accord with the experiments with ammonia masers.

4.3. Mössbauer rotor experiment

Then we consider the case where both the source and the absorber are moving in the laboratory frame. The Mössbauer rotor experiment is based on the recoilless gamma-ray nuclear resonance absorption known as the Mössbauer effect [13-17]. The source and the absorber of gamma ray are placed separately on a rotating rod. Both the quantum energy variation of nuclear states and the Doppler frequency shift of wave propagation cause the frequency shift. A frequency deviation from the resonant-absorption condition exhibits itself with an increase in the scattering of gamma ray, which in turn can be measured by counters fixed at the laboratory or at the rod.

Suppose that the absorber and the source atoms in the Mössbauer rotor experiment rotate about an axis with linear velocities $\mathbf{v}_r$ and $\mathbf{v}_t$, respectively, where the axis is stationary in a laboratory frame which in turn moves at a velocity $\mathbf{v}_0$ with respect to the local-ether frame. In the rotor experiment, the directed path length $\mathbf{R}_t$ from the source to the absorber is always perpendicular both to $\mathbf{v}_r$ and $\mathbf{v}_t$. Moreover, the path length R_t is fixed, while the direction $\hat{R}_t$ is changing with time. Thus $dR_t/dt = 0$ and $d\mathbf{R}_t/dt = \mathbf{v}_{rt}$, where $\mathbf{v}_{rt} = \mathbf{v}_r - \mathbf{v}_t$.

To the second order of normalized speed, the fractional Doppler frequency shift (16) due to the motions of the source and receiver becomes

$$\frac{\triangle f_D}{f_r} = -\frac{d}{cdt}\left(R_t + \frac{1}{c}\mathbf{v}_0 \cdot \mathbf{R}_t\right) = -\frac{\mathbf{v}_0 \cdot \mathbf{v}_{rt}}{c^2}, \tag{25}$$

where the orthogonality $\mathbf{v}_r \cdot \mathbf{R}_t = 0$ has been made use of and the derivative $d\mathbf{v}_0/dt$ is omitted. It is seen that this frequency shift depends on the laboratory velocity $\mathbf{v}_0$. It is noted that due to the path length R_t being a constant, the first-order term of normalized speed vanishes in the preceding formula. Thus, to the third order of normalized speed, the fractional shift $\triangle f_D/f_{0e}$ is identical to the preceding formula of $\triangle f_D/f_r$.

Consider the case where the source and the absorber have the identical rest transition frequency f_0. Thus the fractional frequency shift due to the quantum effect is

$$\frac{\triangle f_Q}{f_0} = \sqrt{1 - (\mathbf{v}_r + \mathbf{v}_0)^2/c^2} - \sqrt{1 - (\mathbf{v}_t + \mathbf{v}_0)^2/c^2} \simeq \frac{1}{2c^2}(-2\mathbf{v}_{rt} \cdot \mathbf{v}_0 - v_r^2 + v_t^2). \tag{26}$$

It is noted that the dot-product term $\mathbf{v}_{rt} \cdot \mathbf{v}_0$ appears again. Consequently, by taking both the Doppler and the quantum effects into account to the second order of normalized speed, the fractional frequency deviation from the resonant absorption is

$$\frac{\delta f}{f_0} = \frac{v_r^2 - v_t^2}{2c^2}. \tag{27}$$

It is noted that the dot-product term $\mathbf{v}_0 \cdot \mathbf{v}_{rt}$ cancels out and hence the frequency deviation is independent of the directions of velocities $\mathbf{v}_r$ and $\mathbf{v}_t$. Furthermore, this frequency deviation is independent of the laboratory velocity $\mathbf{v}_0$.

It is noticed again that a term quite similar to the second-order Doppler effect in (25) has been derived alternatively from a classical approach, aside from the reference frame of the laboratory velocity [14, 16, 17]. Further, a similar cancellation of the terms with $\mathbf{v}_0$ has been accounted for by an alternative approach, where a speed-dependent frequency based on the length contraction is used [16]. Anyway, this fractional frequency deviation has been demonstrated in experiments with various rotation rates and various positions of the source and the absorber at the rod [13-15]. Moreover, it has been found that between the two geostationary counters oriented to detect the gamma rays propagating south- and northward, respectively, there is no substantial difference in the measured frequency deviation [14]. Thus the frequency deviation can be independent of the orientation of counter. Based on these, the local-ether model is also in accord with the Mössbauer rotor experiment.

4.4. Spatial isotropy, Galilean relativity, and their breakdowns

According to the local-ether model, the speed of an earthbound atom is referred to an ECI frame. Thus the quantum energy in an atom is entirely independent of earth's orbital motion around the Sun or whatever. Further, the quantum energy can even be independent of earth's rotation, if the atom speed remains a constant during the rotation. Such a constant-speed condition is met by a geostationary atom, by an atom moving at a fixed velocity with respect to ground at a substantially fixed latitude, or by an atom moving in a circular satellite orbit around the Earth. For an atom satisfying the constant-speed condition, the energies of quantum states and hence the transition frequency between two states are independent of the orientation and position of the Earth in space, whatever the dependence of quantum energy on the speed. This spatial isotropy of transition frequency with respect to earth's motions has been adopted in [3] to account for the hourly and daily frequency stability in geostationary atoms in the Hughes-Drever experiment [18] and for the high synchronism among the various GPS atomic clocks moving in circular orbits [19]. On the other hand, for two atomic clocks moving even at an identical speed but in different directions with respect to the ground, their speeds with respect to an ECI frame tend to be different. Thereby, this isotropy breaks down, as demonstrated in the east-west directional anisotropy in atomic clock rate in the Hafele-Keating experiment [5].

The applicability of the aforementioned spatial isotropy can be extended to the cases where electromagnetic wave as well as matter wave is involved. According to the local-ether model, the results of an earthbound experiment are entirely independent of earth's orbital motion. Further, the results can be independent of earth's rotation, so long as the atom speeds v_e, v_s, u_e, and u_s along with the terms $\mathbf{v}_e \cdot \mathbf{v}_s$ and $\mathbf{a}_e \cdot \mathbf{R}_t$ are invariant under earth's rotation. Thereby, the frequency-variation formula (8), the propagation-range formula (2), and the resonant-absorption condition (11) remain unchanged under earth's rotation and hence the associated phenomena exhibit the spatial isotropy. In the Ives-Stilwell experiment and the ammonia maser, it is seen that the speeds v_e and v_s are invariant under earth's rotation and the acceleration $\mathbf{a}_e$ is zero. Moreover, the terms u_e, u_s, and $\mathbf{v}_e \cdot \mathbf{v}_s$ are invariant under earth's rotation, since $\mathbf{v}_e$, $\mathbf{v}_s$, and $\mathbf{R}_t$ all change in a coordinated way with earth's rotation. Thus the spatial isotropy can be expected in these experiments and has been demonstrated in the hourly and daily stability of the output frequency from ammonia masers [11].

The spatial isotropy with respect to earth's motions can be generalized with a step forward. As the velocities $\mathbf{v}_e$ and $\mathbf{v}_s$ are written in the laboratory frame of velocity $\mathbf{v}_0$, the terms $\mathbf{v}_r \cdot \mathbf{v}_0$, $\mathbf{v}_t \cdot \mathbf{v}_0$, and v_0^2 emerge in the relevant formulas. It has been indicated that the dot-product term $\mathbf{v}_r \cdot \mathbf{v}_0$ or $\mathbf{v}_t \cdot \mathbf{v}_0$ can be common in the Doppler and the quantum effects and then cancels out. Consequently, the results of the experiment are independent of the orientation of the setup with respect to the ground. Thus the experiment possesses a spatial isotropy with respect to the setup orientation as well as to earth's motions. This kind of spatial isotropy is observed in the three experiments examined so far. Thereby, the output frequency from the ammonia maser is independent of the direction of the molecule velocity and hence no observable beat frequency between masers with different orientations can be expected, as demonstrated experimentally in [11]. Moreover, the frequency deviation from the resonant absorption in the Mössbauer rotor experiment can be expected to be stable under earth's rotation and be identical for counters with different orientations, as demonstrated experimentally in [14].

Further, the squared term v_0^2/c^2 in some frequency formulas cancels out or is too small to detect. As a consequence, the experimental results become independent of the laboratory velocity $\mathbf{v}_0$ and hence comply with Galilean relativity, in spite of the restriction on the reference frame of the particle and the propagation velocities. Thus, when the whole experimental setup is put on a vehicle moving smoothly with respect to the ground, the measurement results can be independent of the ground velocity of the vehicle, as well as of the setup orientation and earth's motions. Thereby, the spatial isotropy is generalized to the compliance with Galilean relativity. In this way, it is seen that the Mössbauer rotor experiment complies with Galilean relativity. Moreover, Galilean relativity can also be observed in the Ives-Stilwell experiment and in the output frequency from the ammonia maser, as the minute squared term is difficult to detect. Anyway, these experiments preserve the spatial isotropy with respect to the setup orientation and earth's motions.

However, when the measurement precision is improved such that higher-order effects can be detected, some minute terms may emerge to make Galilean relativity or even the spatial isotropy break down. The just-mentioned squared term v_0^2/c^2 in the Ives-Stilwell experiment and the ammonia maser is an example. For another, we reexamine the Mössbauer rotor experiment to a higher order. In order to derive the Doppler effect to the third order of normalized speed, the second-order propagation-range formula (2) is needed, which for the Mössbauer rotor experiment becomes

$$R(t) = R_t \left\{ 1 + \frac{1}{c} u_0 + \frac{1}{2c^2} [u_0^2 + (v_r^2 + v_0^2 + 2\mathbf{v}_r \cdot \mathbf{v}_0) + \mathbf{a}_e \cdot \mathbf{R}_t] \right\}. \tag{28}$$

For a rotation at a fixed angular velocity, the terms of v_r and $\mathbf{a}_e \cdot \mathbf{R}_t$ as well as v_0 are constants during rotation. Thus the time rate of change of the propagation range R is given by

$$\frac{dR(t)}{dt} = R_t \frac{d}{dt} \left\{ \frac{1}{c} u_0 + \frac{1}{2c^2} [u_0^2 + 2\mathbf{v}_r \cdot \mathbf{v}_0] \right\}. \tag{29}$$

Then, to the third order of normalized speed, the fractional Doppler frequency shift due to wave propagation can be given from (16) as

$$\frac{\triangle f_D}{f_0} = -\frac{dR}{cdt} = -\frac{1}{c^2} \mathbf{v}_{rt} \cdot \mathbf{v}_0 - \frac{1}{c^3} [(\mathbf{v}_{rt} \cdot \mathbf{v}_0) u_0 - v_r v_{rt} u_0], \tag{30}$$

where we have made use of $d\mathbf{v}_r/dt = -\hat{R}_t v_r v_{rt}/R_t$ and of $\triangle f_D/f_0 = \triangle f_D/f_r$ to the third order. It is seen that $\triangle f_D = 0$ when $v_0 = 0$.

By taking both the Doppler and the quantum effects into account to the third order of normalized speed, the fractional frequency deviation from the resonant absorption is

$$\frac{\delta f}{f_0} = \frac{1}{2c^2} (v_r^2 - v_t^2) - \frac{1}{c^3} (\mathbf{v}_{rt} \cdot \mathbf{v}_0 - v_r v_{rt}) u_0. \tag{31}$$

It is noted that the dot-product term $\mathbf{v}_{rt} \cdot \mathbf{v}_0$ survives in the third-order effect, although it cancels out in the second-order one. The presence of this term then makes the spatial isotropy with respect to the setup orientation and hence Galilean relativity break down, although it is quite small in magnitude. Accordingly, the frequency deviations tend to be different as measured in counters with different orientations with respect to the ground, when they can be measured to the third order. Thereby, a directional anisotropy in the frequency deviation is predicted. On the other hand, the spatial isotropy with respect to earth's rotational and orbital motions is preserved.

5. Conclusion

Based on the local-ether wave equation for matter wave, the energies of quantum states and hence the state transition frequency in an atom decrease with the atom speed by the mass-variation factor, where the speed is referred to the associated local-ether frame. Further, for the situation where the source and the receiver atoms move at different velocities, the local-ether resonant-absorption condition is presented by taking both the Doppler frequency shift for electromagnetic wave and the quantum energy variation of matter wave into account. It is shown that the resonant-absorption condition accounts for the Ives-Stilwell experiment, the output frequency from ammonia masers, and the Mössbauer rotor experiment in a consistent way.

Based on the local-ether model, it is evident that the phenomena due both to electromagnetic and matter waves do not at all depend on earth's orbital motion around the Sun or others. This immediately accounts for the null effect of earth's orbital motion in various earthbound phenomena. Further, for a geostationary experimental setup, the results can be independent of earth's rotation. This accounts for the spatial isotropy found in the stability of frequency in the Hughes-Drever experiment and the ammonia maser. The spatial isotropy also holds for the atomic clocks onboard earth's satellites moving in circular orbits and hence accounts for the high synchronism among the various GPS atomic clocks. Further, in the formula of the Doppler or the quantum effect, it is seen that the major term associated with the laboratory velocity is a dot product between this velocity and that of the emitting or absorbing atom. This term appears both in the Doppler frequency shift and the transition frequency variation and then cancels out. Thus the experimental results become invariant with respect to the setup orientation, as well as to earth's motions. This spatial isotropy has been demonstrated in the ammonia maser where the output frequency is independent of the direction of the molecule velocity and in the Mössbauer rotor experiment where the frequency deviation from the resonant absorption is independent of the orientation of the counter. Further, the frequency deviation is independent of the laboratory velocity and hence the Mössbauer rotor experiment complies with Galilean relativity.

However, the aforementioned dot-product term with the laboratory velocity may survive in some formulas. Thus the spatial isotropy with respect to the setup orientation and hence Galilean relativity break down. This breakdown is in accord with the east-west directional anisotropy in atomic clock rate demonstrated in the Hafele-Keating experiment. By examining the resonant-absorption condition in the Mössbauer rotor experiment to the third order, such a breakdown is also found. Thereby, it is expected that the frequency deviation tends to be different as measured in counters with different orientations. This predicted directional anisotropy in the frequency deviation may provide a mean to test the local-ether model, if the measurement precision can be raised to the third order of normalized speed.

References

[1] H.E. Ives and G.R. Stilwell, *J. Opt. Soc. Am.* **28**, 215 (1938); *J. Opt. Soc. Am.* **31**, 369 (1941).

[2] A.P. French, *Special Relativity* (Chapman & Hall, New York, 1968), ch. 5.

[3] C.C. Su, *Eur. Phys. J. B* **24**, 231 (2001).

[4] C.C. Su, *Eur. Phys. J. C* **21**, 701 (2001); *Europhys. Lett.* **56**, 170 (2001).

[5] J.C. Hafele and R.E. Keating, *Science* **177**, 166 (1972); **177**, 168 (1972).

[6] H.E. Ives, *J. Opt. Soc. Am.* **27**, 263 (1937).

[7] R.J. Kennedy and E.M. Thorndike, *Phys. Rev.* **42**, 400 (1932).

[8] C.C. Su, in *Bull. Am. Phys. Soc.* (Apr. 2001), p. 99.

[9] L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields* (Pergamon, New York, 1975), sect. 48.

[10] H.I. Mandelberg and L. Witten, *J. Opt. Soc. Am.* **52**, 529 (1962).

[11] J.P. Cedarholm, G.F. Bland, B.L. Havens, and C.H. Townes, *Phys. Rev. Lett.* **1**, 342 (1958).

[12] J.P. Cedarholm and C.H. Townes, *Nature* **181**, 1350 (1959).

[13] H.J. Hay, J.P. Schiffer, T.E. Cranshaw, and P.A. Egelstaff, *Phys. Rev. Lett.* **4**, 165 (1960).

[14] D.C. Champeney, G.R. Isaak, and A.M. Khan, *Phys. Lett.* **7**, 241 (1963).

[15] K.C. Turner and H.A. Hill, *Phys. Rev.* **134**, B252 (1964).

[16] M. Ruderfer, *Phys. Rev. Lett.* **5**, 191 (1960); **7**, 361 (1961).

[17] J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), ch. 11.

[18] M.P. Haugan and C.M. Will, *Phys. Today* **40**, 69 (1987).

[19] T. Logsdon, *The NAVSTAR Global Positioning System* (Van Nostrand Reinhold, New York, 1992), chs. 2 and 11.