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A local-ether wave equation and speed-dependent mass and quantum energy

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Abstract. The east-west directional anisotropy in clock rate observed in the Hafele-Keating experiment with circumnavigation atomic clocks is commonly ascribed to the special relativity. In this investigation, based on the local-ether wave equation, an entirely different interpretation of this anisotropy is presented by showing that the clock-rate variation can originate from an intrinsic quantum property of the atom. For a harmonic-like wavefunction, the local-ether wave equation leads to a first-order time evolution equation similar to Schrödinger's equation. However, the time derivative incorporates a speed-dependent factor similar to that in the Lorentz mass-variation law. Consequently, the quantum energy, the transition frequency, and hence the atomic clock rate decrease with the atom speed by this speed-dependent mass-variation factor. According to the local-ether model, the speed is referred specifically to a geocentric or heliocentric inertial frame for an earthbound or interplanetary clock, respectively. It is shown that this restriction on reference frame is actually in accord with the various experimental results of the anisotropy and the clock-rate difference in the Hafele-Keating experiment, the synchronism and the clock-rate adjustment in GPS (global positioning system), and of the spatial isotropy in the Hughes-Drever experiment. Moreover, the switching of the unique reference frame is in accord with the frequency-shift formulas adopted in earthbound and interplanetary spacecraft microwave links. Meanwhile, the local-ether model predicts a constant deviation in frequency shift from the calculated result reported in an interplanetary spacecraft link. This discrepancy then provides a means to test the local-ether wave equation.

PACS. 03.65.-w Quantum mechanics – 03.65.Pm Relativistic wave equations – 04.60.-m Quantum gravity

1 Introduction

From the experiment of circumnavigation atomic clocks by Hafele and Keating in 1971, it has been verified that the tick rate of an atomic clock depends on its speed. In this experiment, cesium atomic clocks were flown around the Earth, first eastward and then westward. After each trip, the circumnavigation clocks were compared with a geostationary one. It has been found that the atomic clocks flying westward tick at a faster rate than a geostationary one, while they tick at a slower rate when flying eastward [1,2]. Quantitatively, it has been found that when the atomic clock is moving at a speed v, its tick rate slows down by a factor of $\sqrt{1 - v^2/c^2}$, where c is the speed of light. It is noted that this factor is just the inverse of the Lorentz mass-variation factor which in turn has been demonstrated in the charge-mass ratio of high-energy electrons in the famous Bucherer's experiment [3].

For a long time, the overwhelmingly dominant interpretation of this speed-dependent variation in clock rate is ascribed [1,2] to a second-order Doppler effect in the propagation of electromagnetic wave between a transmitter and a receiver in relative motion, which in turn is derived in Einstein's original paper on the special relativity from a kinematical viewpoint based on the Lorentz transformation of space and time [4]. In this investigation, an entirely different interpretation of the speed-dependent atomic clock rate is presented by showing that the clock rate together with its speed-dependence can be an intrinsic quantum property of matter wave bounded in the associated atom.

Recently, we have presented the local-ether model of wave propagation [5]. That is, electromagnetic wave can be viewed as to propagate *via* a medium like the ether. However, the ether is not universal. It is supposed that in the region under sufficient influence of the gravity due to the Earth, the Sun, or another celestial body, there forms a local ether which in turn moves with the gravitational potential of the respective body. Each individual local ether is finite in extent and may be wholly immersed in another local ether of larger extent. Thus the local ethers may form a multiple-level hierarchy. For earthbound wave, the medium is the earth local ether which is stationary in

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an ECI (earth-centered inertial) frame, while the sun local ether for interplanetary wave is stationary in a heliocentric inertial frame. Consequently, for a geostationary observer, an earthbound wave depends on earth's rotation but is entirely independent of earth's orbital motion around the Sun or whatever, while an interplanetary wave depends on the orbital motion around the Sun as well as on the rotation. This local-ether model has been adopted to account for a wide variety of propagation phenomena, particularly the GPS (global positioning system) Sagnac correction, the time comparison by intercontinental microwave link, and the interplanetary radar echo time [5].

Further, matter wave is supposed to follow the local-ether model and then be governed by a wave equation. Under the condition of low particle speed, the local-ether wave equation leads to a first-order time evolution equation, which is similar to the famous Schrödinger's equation [6,7]. From the evolution equation, the velocity and the acceleration of a particle can be derived in a quantum-mechanical approach. Under the influence of the electric scalar and the gravitational potentials, a unified quantum theory of the electromagnetic and the gravitational forces in conjunction with the identity of inertial and gravitational mass has been derived [6,7].

In this investigation, the restriction on particle speed is removed. Then the corresponding evolution equation, particle velocity, and speed-dependent mass are derived. Thereafter, the effects of the speed-dependent mass and the gravitational potential on the energies of quantum states of matter wave bounded in atom are explored and the associated speed- and gravitation-dependent transition frequency is used to account for the clock-rate variations in the Hafele-Keating experiment, GPS, earthbound and interplanetary spacecraft microwave links, and in the Hughes-Drever experiment in a consistent way.

2 Local-ether wave equation and evolution equation

It is supposed that matter wave associated with a particle follows the local-ether model. Under the electrical scalar potential Φ due to charged particles and the gravitational potential $\Phi_{\rm g}$ due to a celestial body, the matter wave Ψ associated with a particle of charge q and natural frequency ω_0 is supposed to be governed by the local-ether wave equation proposed to be

$$\left\{\frac{1}{n_{\rm g}}\nabla^2 - \frac{n_g}{c^2}\frac{\partial^2}{\partial t^2}\right\}\Psi(\mathbf{r},t) = \frac{\omega_0^2}{c^2}\left\{1+\frac{2}{\hbar\omega_0}q\Phi(\mathbf{r},t)\right\}\Psi(\mathbf{r},t), \quad (1)$$

where the *gravitational index* $n_{\rm g} = 1 + 2\Phi_{\rm g}/c^2$, $\Phi_{\rm g}(\mathbf{r}) = GM/r$, G is the gravitational constant, and r $(= |\mathbf{r}|)$ is the radial distance away from the center of the celestial body of mass M. The position vector $\mathbf{r}$ and the time derivative in this wave equation are referred uniquely to the local-ether frame associated with the celestial body, which is a geocentric or a heliocentric inertial frame for an earthbound or an interplanetary particle, respectively. This feature is simply analogous to the fact that the position vector and the time derivative in the wave equation governing the mechanical wave on a violin string are referred specifically to the frame in which the violin is stationary. This local-ether wave equation made its debut in [7].

In the absence of the potentials, the local-ether wave equation reduces to a form looks like the Klein-Gordan equation for a free particle [8]. Further, if the natural frequency is zero, the wave equation reduces to that for electromagnetic wave. From the wave equation it is seen that if the potentials and the spatial variation of the wavefunction are weak, the wavefunction tends to oscillate at the natural frequency ω_0. Under the influence of the potentials, which are functions of space, the spatial variation of the wavefunction tends to change. Accordingly, its temporal variation tends to increase.

Suppose that the spatial variation of the wavefunction Ψ is close to a space harmonic $e^{i\mathbf{k}\cdot\mathbf{r}}$, where $\mathbf{k}$ is a constant known as the propagation vector. Thus the wavefunction can be given as $\Psi(\mathbf{r},t) = \tilde{\psi}(\mathbf{r},t)e^{i\mathbf{k}\cdot\mathbf{r}}$ and then its Laplacian becomes

$$\nabla^2\Psi(\mathbf{r},t) = \left\{\nabla^2\tilde{\psi}(\mathbf{r},t) + i2\mathbf{k}\cdot\nabla\tilde{\psi}(\mathbf{r},t) - k^2\tilde{\psi}(\mathbf{r},t)\right\}e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (2)$$

Thereby, the local-ether wave equation becomes

$$\left\{\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}\tilde{\psi}(\mathbf{r},t) = \frac{\omega^2}{c^2}\tilde{\psi}(\mathbf{r},t) + \frac{2\omega_0}{\hbar c^2}q\Phi\tilde{\psi}(\mathbf{r},t) - i2\mathbf{k}\cdot\nabla\tilde{\psi}(\mathbf{r},t), \quad (3)$$

where the gravitational potential is omitted for simplicity and its effect will be considered later. The angular frequency ω combines the natural frequency ω_0 and the propagation constant k as

$$\omega^2 = \omega_0^2 + c^2k^2. \quad (4)$$

The spatial variation of $\tilde{\psi}$ should be much weaker than that of Ψ. Further, if the potential Φ as well as the spatial rate of change of $\tilde{\psi}$ is relatively weak, the temporal variation of $\tilde{\psi}$ or Ψ can be expected to be close to the time harmonic $e^{-i\omega t}$. Thus the wavefunction can be given as $\tilde{\psi}(\mathbf{r},t) = \psi(\mathbf{r},t)e^{-i\omega t}$ and then its second time derivative becomes

$$\frac{\partial^2}{\partial t^2}\tilde{\psi}(\mathbf{r},t) = \left\{\frac{\partial^2}{\partial t^2}\psi(\mathbf{r},t) - i2\omega\frac{\partial}{\partial t}\psi(\mathbf{r},t) - \omega^2\psi(\mathbf{r},t)\right\}e^{-i\omega t}. \quad (5)$$

As the temporal variation of ψ is relatively weak, the second derivative $\partial^2\psi/\partial t^2$ can be neglected.

Thereby, the local-ether wave equation can be approximated to the first-order time evolution equation

$$\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \mathrm{i}\frac{c^2}{2\omega}\nabla^2\psi(\mathbf{r},t) - \mathrm{i}\frac{\omega_0}{\hbar\omega}q\Phi\psi(\mathbf{r},t) - \frac{c^2}{\omega}\mathbf{k}\cdot\nabla\psi(\mathbf{r},t). \tag{6}$$

Note that the major term with $\omega^2\psi$ cancels out. The wavefunction ψ should have a greatly reduced variation in space and time, since the space- and time-harmonic term $\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\mathrm{e}^{-\mathrm{i}\omega t}$ has been factored out from the wavefunction Ψ, which in turn is close to this harmonic. The wavefunction Ψ and the reduced wavefunction ψ are related to each other as

$$\Psi(\mathbf{r},t) = \psi(\mathbf{r},t)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\mathrm{e}^{-\mathrm{i}\omega t}. \tag{7}$$

It is noted that by virtue of the natural frequency, such a harmonic-like wavefunction Ψ becomes dispersive as represented by the dispersion relation (4).

According to the wave equation or the evolution equation in conjunction with the associated initial conditions, the wavefunction Ψ or the reduced wavefunction ψ as a function of space and time can be determined. The evolution equation involves only first-order time derivative and is simpler. Further, some general formulas representing physical quantities, such as velocity and acceleration, of the associated particle can be derived from the evolution equation, as in the treatment based on Schrödinger's equation. However, the applicability of the evolution equation is more restricted, since the spatial and the temporal variations in the wavefunction Ψ tend to change with time and hence the propagation vector $\mathbf{k}$ and the angular frequency ω of the factored-out harmonic could need to be updated frequently to maintain the accuracy.

3 Particle velocity and speed-dependent mass

As in quantum mechanics, the physical quantity represented by an operator O is supposed to be given by the expectation value of the operator evaluated in terms of the reduced wavefunction ψ as $\langle O\rangle = \int \psi^* O\psi \mathrm{d}\mathbf{r}$. Thus the velocity of a particle is given by the time derivative of the expectation value of its position vector as $\mathbf{v} = \mathrm{d}\langle\mathbf{r}\rangle/\mathrm{d}t$, where the position vector $\mathbf{r}$ and hence the velocity $\mathbf{v}$ are referred to the local-ether frame. By expanding the local-ether-frame time derivative of expectation value of the time-independent operator of the position vector according to the evolution equation (6), it can be shown that the particle velocity is given by the expectation value of the del operator as

$$\mathbf{v} = -\mathrm{i}\frac{c^2}{\omega}\langle\nabla\rangle + \frac{c^2}{\omega}\mathbf{k}, \tag{8}$$

where the wavefunction is supposed to be normalized such that $\int \psi^*\psi\mathrm{d}\mathbf{r} = 1$. It is easy to see that the expectation value $\langle\nabla\rangle$ in ψ is related to the expectation value $\langle\nabla\rangle_\Psi$ in Ψ as

$$\langle\nabla\rangle_\Psi = \langle\nabla\rangle + \mathrm{i}\mathbf{k}, \tag{9}$$

where $\langle O\rangle_\Psi = \int \Psi^* O\Psi\mathrm{d}\mathbf{r}$ denotes the expectation value evaluated in terms of wavefunction Ψ. Thereby, the velocity can be given by

$$\mathbf{v} = -\mathrm{i}\frac{c^2}{\omega}\langle\nabla\rangle_\Psi. \tag{10}$$

It is noted that the particle velocity with respect to the local-ether frame is proportional to the spatial rate of change of the wavefunction Ψ (rather than of the reduced wavefunction ψ) and to the inverse of the temporal rate of change of Ψ.

When the particle speed is low enough, the propagation vector $\mathbf{k}$ in the factored-out harmonic can be chosen to be zero and hence the angular frequency ω becomes the natural frequency ω_0. Thereby, the evolution equation for a slowly-moving particle reduces to

$$\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \mathrm{i}\frac{c^2}{2\omega_0}\nabla^2\psi(\mathbf{r},t) - \mathrm{i}\frac{1}{\hbar}q\Phi\psi(\mathbf{r},t). \tag{11}$$

From this evolution equation, it has been pointed out in [7] that the acceleration of a low-speed particle under the influence of the electric scalar potential is given by

$$\mathbf{a} = -\frac{c^2}{\hbar\omega_0}q\nabla\Phi, \tag{12}$$

where the acceleration due to the gravitational potential ($\nabla\Phi_g$) is omitted. It is noted that the acceleration is inversely proportional to ω_0. Then, from Newton's second law of motion and the well-known electrostatic force $\mathbf{F} = -q\nabla\Phi$, it has been pointed out that the natural frequency ω_0 multiplied by the constant $(\hbar/c^2)$ is just the inertial mass of the particle under the influence of the electric scalar potential Φ, as the particle is at rest or moves at a low speed with respect to the local-ether frame. That is, the natural frequency ω_0 is related to the rest mass m_0 of the particle as

$$m_0 = \frac{\hbar}{c^2}\omega_0. \tag{13}$$

This simple frequency-mass relation unveils the physical origin of mass as a wave motion.

When the spatial variation of wavefunction Ψ is quite close to the harmonic $\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}$, the expectation value $\langle\nabla\rangle$ approaches zero and the expectation value $\langle\nabla\rangle_\Psi$ approaches $\mathrm{i}\mathbf{k}$. Thus the velocity formula (10) reduces to the form of

$$\mathbf{k} = \frac{\omega}{c^2}\mathbf{v}. \tag{14}$$

This propagation vector-velocity relation is valid even when the particle speed is high. It is noted that the propagation vector $\mathbf{k}$ is linearly proportional to the velocity $\mathbf{v}$ with the angular frequency ω divided by c^2 as the ratio.

This formula is identical to that derived from the dispersion relation of a wave packet by evaluating its group velocity discussed in [8]. Thus, for such a harmonic-like wave packet Ψ, the expectation value $\langle\nabla\rangle_\Psi$ is proportional to the group velocity. That is, the spatial rate of change and the group velocity of a wave packet are closely related properties of wave motion.

Further, on substituting the preceding relation back into the dispersion relation (4), the angular frequency can be expressed in terms of the particle speed v as

$$\omega = \omega_0 \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{15}$$

where speed v is referred to the local-ether frame. It is seen that the angular frequency is the natural frequency times the speed-dependent factor. According to this speed-dependent frequency and the frequency-mass relation (13), the speed-dependent mass m is defined in terms of the rest mass m_0 as

$$m = m_0 \frac{1}{\sqrt{1 - v^2/c^2}} \cdot \tag{16}$$

Then, in terms of the speed-dependent mass m, the angular frequency and the propagation vector of the matter wave Ψ can be given by

$$\hbar\omega = mc^2 \tag{17}$$

and

$$\hbar\mathbf{k} = m\mathbf{v}, \tag{18}$$

where the propagation vector formula (14) has been made used of. It is noted that the preceding three formulas of speed-dependent mass, angular frequency, and propagation vector are just the famous de Broglie postulates for matter wave in conjunction with the Lorentz mass-variation law, if the reference frame of the particle velocity $\mathbf{v}$ is ignored. The speed-dependent wavelength has been demonstrated in the Davisson-Germer experiment, the double-slit diffraction, and the Sagnac effect of matter wave. In these earthbound experiments, the particle velocity is expected to be referred to an ECI frame. As the speeds of the associated particles are much higher than the linear speed due to earth's rotation, the local-ether model is substantially in accord with those experiments adopting the laboratory frame.

4 Speed-dependent quantum energy

In terms of the speed-dependent mass m, the time evolution equation (6) governing the reduced wavefunction ψ can be rewritten as

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + \frac{m_0}{m}q\Phi\psi(\mathbf{r},t) - \mathrm{i}\frac{\hbar^2}{m}\mathbf{k}\cdot\nabla\psi(\mathbf{r},t), \tag{19}$$

where the time derivative is referred to the local-ether frame. Note that the whole interaction term decreases with increasing mass as $(m_0/m)q\Phi$. Ignoring the reference frame, this evolution equation reduces to Schrödinger's equation when $k = 0$ and hence $m = m_0$.

Consider an atom, a molecule, or an ion which is moving at a velocity $\mathbf{v}_\mathrm{a}$ with respect to the local-ether frame. It is expected that the electric scalar potential Φ due to the nucleus and electrons in an atom will move with this atom. Accordingly, the potential is stationary in the atom frame with respect to which the atom is stationary, while it is moving in the local-ether frame. Under Galilean transformations, the potential moving with the atom can be written as $\Phi(\mathbf{r})$ or as $\Phi(\mathbf{r}-\mathbf{v}_\mathrm{a}t)$, where the position vector $\mathbf{r}$ is referred to the atom or to the local-ether frame, respectively. The average value of the velocity of a particle (electron or nucleon) bounded in the atom should be identical to the atom velocity $\mathbf{v}_\mathrm{a}$; otherwise, the particle tends to escape from the atom. Thus the spatial and temporal variation of the wavefunction of the bounded particle can be expected to be close to the harmonic $\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\mathrm{e}^{-\mathrm{i}\omega t}$ and then the reduced wavefunction is governed by the preceding evolution equation, where $\mathbf{k} = m\mathbf{v}_\mathrm{a}/\hbar$, $m = m_0/\sqrt{1 - v_\mathrm{a}^2/c^2}$, the potential is given by $\Phi(\mathbf{r}-\mathbf{v}_\mathrm{a}t)$, and the position vector $\mathbf{r}$ is referred to the local-ether frame.

We next go on to rearrange the evolution equation to express it in the atom frame, instead of the local-ether frame. To begin with, it is noted that the last term in (19) can be written as $(\hbar^2/m)\mathbf{k}\cdot\nabla = \hbar\mathbf{v}_\mathrm{a}\cdot\nabla$. The time derivative observed in the atom frame, denoted as $(\partial/\partial t)_\mathrm{a}$, is generally different from the derivative $\partial/\partial t$ observed in the local-ether frame. Based on Galilean transformations, the time derivatives $\partial/\partial t$ and $(\partial/\partial t)_\mathrm{a}$ are understood to be taken under constant $\mathbf{r}$ and $(\mathbf{r}-\mathbf{v}_\mathrm{a}t)$, respectively, as $\mathbf{r}$ is referred to the local-ether frame. It is known that for an arbitrary function f of space and time [19],

$$\left(\frac{\partial f}{\partial t}\right)_\mathrm{a} = \frac{\partial f}{\partial t} + \mathbf{v}_\mathrm{a}\cdot\nabla f. \tag{20}$$

Thereby, for a particle bounded in a moving atom, the time evolution equation observed in the atom frame becomes

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + \frac{m_0}{m}q\Phi(\mathbf{r})\psi(\mathbf{r},t), \tag{21}$$

where the position vector $\mathbf{r}$ and hence the time derivative are referred to the atom frame, instead of the local-ether frame. It is noted that the mass associated with the Laplacian is the speed-dependent mass, instead of the rest mass. Moreover, the interaction term incorporates an extra multiplying term of m_0/m, inverse to the mass-variation factor. It is also noted that the time evolution equation as well as the potential is independent of the movement of atom if the equation is observed in the atom frame and the mass variation is neglected, as assumed so tacitly in common practice. In other words, in applying Schrödinger's equation with a stationary potential to deal with quantum states in an atom, one has actually adopted the atom frame.

As the matter wave is bounded in an atom, the temporal variation of the reduced wavefunction itself is supposed to be time harmonic as $\psi(\mathbf{r},t) = \psi(\mathbf{r})\mathrm{e}^{-\mathrm{i}\tilde{\omega}t}$, as a consequence of resonance. Then the quantity $\hbar(\omega + \tilde{\omega})$ will be the energy of quantum state of the bounded matter wave. The minor part $\hbar\tilde{\omega}$ tends to be different in a different quantum state, while the major part $\hbar\omega$ is identical in all the states. Then the preceding evolution equation leads to the time-independent Helmholtz equation

$$-\frac{\hbar^2}{2m_0}\nabla^2\psi(\mathbf{r}) + q\Phi(\mathbf{r})\psi(\mathbf{r}) = \frac{m}{m_0}\hbar\tilde{\omega}\psi(\mathbf{r}). \tag{22}$$

This equation looks like the time-independent equation derived from (11) for a particle moving slowly in the local-ether frame, in spite that the position vector $\mathbf{r}$ here is referred specifically to the atom frame and that the mass-variation factor m/m_0 connects to $\hbar\tilde{\omega}$. Consequently, the solutions for the eigenfunction ψ and the eigenvalue $\hbar\tilde{\omega}(m/m_0)$ of this equation in the atom frame will be independent of the atom speed v_a. Accordingly, as compared to that of a stationary atom, the minor energy $\hbar\tilde{\omega}$ of each quantum state will decrease with the inverse of the mass-variation factor, when the atom is moving at speed v_a with respect to the local-ether frame. Meanwhile, the major energy $\hbar\omega$ for a moving atom increases by this factor.

The frequency of light emitted from or absorbed by an atom is known to be equal to the transition frequency which in turn is proportional to the difference between the energies of two involved quantum states. As the major energy $\hbar\omega$ is identical in all the states, its effect on the transition frequency cancels out. Thus the transition frequency f is determined by the minor energy $\hbar\tilde{\omega}$ and then is inversely proportional to the speed-dependent mass m. Precisely, the transition frequency decreases with increasing atom speed by the mass-variation factor as

$$f = f_0\sqrt{1 - v_\mathrm{a}^2/c^2}, \tag{23}$$

where f_0 is the rest transition frequency of the atom when it is stationary in the local-ether frame, the atom speed v_a is referred specifically to the local-ether frame, and the transition frequencies are observed in the atom frame such that the Doppler effect due to the relative motion between source and receiver vanishes. Speed-dependent frequency of this form (but of different physical meaning and reference frame) was first introduced by Fitzgerald, Lorentz, and Larmor before the advent of the special relativity [10] and was later derived by assuming the length contraction [11] or the time dilation [12]. According to the local-ether model, the transition frequency of an earthbound atom depends on its speed with respect to a geocentric inertial frame. Meanwhile, for an atom onboard an interplanetary spacecraft, the transition frequency depends on the speed with respect to a heliocentric one.

5 Reexamination of experiments with atomic clock rates

The clock rate of an atomic clock is determined by the transition frequency between two quantum states. If the quantum states is directly due to the electric scalar potential, then the clock rate definitely slows down by the mass-variation effect. However, the quantum states can be modified by some other mechanisms, such as the hyperfine splitting due to the spin-spin interaction between electron and nucleon in the cesium atomic clock [13] or the hydrogen maser [14] discussed in this section. Although the spin can not be treated by the local-ether wave equation, it is expected that this interaction leads to an equation similar to (22), except a modification in the interaction term $q\Phi$. Thus the transition frequency and hence the clock rate also decrease by the speed-dependent mass-variation factor. On the other hand, different dependences of transition frequency on mass or speed caused by some other interactions are not precluded. As the precision in the experiments to detect the speed-dependent effect is high, the gravitational effect often becomes significant and hence is also taken into consideration. Thereafter, the derived gravitation- and speed-dependent transition frequency formula is used to account for the atomic clocks in the Hafele-Keating experiment, GPS, and in earthbound and interplanetary spacecraft microwave links, which are commonly ascribed to the general and the special relativity.

5.1 Gravitation- and speed-dependent transition frequency

If the gravitational effect is taken into account, the local-ether wave equation (1) leads to the algebraic equation for a matter wave bounded in a moving atom as

$$(\omega + \tilde{\omega})^2 = \omega_0^2\left\{1 + \left[\left(\frac{1}{n_\mathrm{g}} - 1\right) - \frac{c^2}{n_\mathrm{g}^2\omega_0^2}\left\langle\nabla^2\right\rangle_\Psi + \frac{1}{n_\mathrm{g}}\frac{2}{\hbar\omega_0}\left\langle q\Phi\right\rangle\right]\right\}, \tag{24}$$

where wavefunction $\Psi(\mathbf{r},t) = \psi(\mathbf{r})\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\mathrm{e}^{-\mathrm{i}(\omega+\tilde{\omega})t}$, $k = mv_\mathrm{a}/\hbar$, and the wavefunction is supposed to be normalized. Then an expansion like (2) leads to $\left\langle\nabla^2\right\rangle_\Psi = -k^2 + \left\langle\nabla^2\right\rangle$, where we have made use of that a bounded wavefunction ψ is real and hence the expectation value $\langle\nabla\rangle = 0$. When the gravitational potential is weak, the perturbation method can be applied. Thereby, the angular frequency can be evaluated in terms of the corresponding wavefunction in the absence of gravitational potential [15]. Consider the case of atomic clocks which are associated with the spin-spin interaction between wavefunctions of identical spatial distribution but of different spin states. Then the state transition is associated only with the interaction $q\Phi$.

Suppose that both the potential Φ and the spatial rate of change of Ψ as well as the potential Φ_g are weak. Then, by evaluating the square root of the right-hand side of the preceding frequency formula with binomial expansion to the second order and retaining only those terms associated with the interaction $q\Phi$, it can be shown that due to the gravitational and the speed-dependent mass-variation effects, the transition frequency f decreases as

$$f = f_0 \left(1 - \frac{\Phi_g}{c^2} - \frac{v_a^2}{2c^2}\right) + f_{ex}, \tag{25}$$

where f_0 is the rest transition frequency of the atom when it is stationary in the local-ether frame and at a zero gravitational potential and f_{ex} denotes extra minor contributions from the spatial variation of ψ and the second-order effect of the interaction. As the atom speed v_a has been supposed to be much lower than c, it is seen that the speed-dependence in the preceding frequency formula is in accord with (23). When the restriction on the atom speed is removed, the transition frequency is expected to given by $f = f_0\sqrt{1 - v_a^2/c^2}(1 - \Phi_g/c^2)$. Quantitatively, frequency f_0 is given by

$$f_0 = \frac{q}{2\pi\hbar}\int (\psi_a \Phi \psi_a - \psi_b \Phi \psi_b) d\mathbf{r}, \tag{26}$$

where ψ_a and ψ_b are the reduced wavefunctions of the two quantum states involved in the transition. Based on the perturbation method, ψ_a and ψ_b can be replaced with the corresponding solutions of the Helmholtz equation (22), where the gravitational potential is omitted. Recall that these solutions are independent of the atom speed. It is noted that according to the local-ether wave equation, the gravitational redshift as well as the speed-dependent variation in transition frequency is associated with quantum nature of bounded matter wave.

5.2 Clock-rate difference in Hafele-Keating experiment

Consider the atomic clocks onboard a circumnavigating aircraft in the Hafele-Keating experiment. According to the local-ether model, the speed in the mass-variation factor is referred specifically to an ECI frame, rather than to the ground or any other frame. Thus this speed is dependent on earth's rotation and latitude, but is entirely independent of the orbital motion of the Earth around the Sun or whatever. When the aircraft is flying westward, the atomic clock tends to have a lower speed with respect to an ECI frame than a geostationary one. Consequently, the atomic clock flying westward ticks at a faster rate than a geostationary one, while the one flying eastward ticks at a slower rate. Thereby, earth's rotation leads to an east-west directional anisotropy in the atomic clock rate.

Quantitatively, according to the local-ether model, the speed that determines the tick rate of the atomic clock onboard an aircraft flying at a ground speed v_f east- or westward is given by the sum $v_E + v_f$, where $v_E = 464 \times \cos\theta_l$ m/sec is the speed of ground due to earth's rotation, θ_l is the latitude, and v_f is positive when the clock is flown eastward and negative when westward. By taking both the speed- and the gravitation-dependent effects into account, the fractional difference in transition frequency between the flying and the geostationary atomic clocks at identical latitude is given by the local-ether model as

$$\frac{\triangle f}{f_0} = -\frac{2v_E v_f + v_f^2}{2c^2} + \frac{MG}{c^2 R_E^2} h_a, \tag{27}$$

where M and R_E are the mass and the radius of the Earth, respectively, and h_a is the altitude of the aircraft.

In the Hafele-Keating experiment, based on the recorded flight data of aircraft ground speed, latitude, altitude, and of clock time, the difference in clock time between the flying and the geostationary clocks was calculated by numerical integration for each circumnavigation trip in either direction. The formula for calculation presented in [1] is identical to (27), although it is based on an entirely different theory of the special and the general relativity. Thereby, the calculated clock-time difference for each circumnavigation trip east- or westward is −40 or 275 ns, respectively, which has been found to agree with the experimental results [1]. Thus it is evident that the local-ether model is in accord with the east-west directional anisotropy in atomic clock rate demonstrated in the Hafele-Keating experiment.

5.3 Clock-rate adjustment in GPS

The GPS employs about 24 half-synchronous satellites carrying highly precise and synchronized cesium and rubidium atomic clocks around six nearly circular orbits [13]. It is known that the various GPS atomic clocks keep a high synchronism within a few ns among themselves over a long duration of time (one day) before routine clock correction [13]. As the orbits are nearly circular, the various GPS atomic clocks move at nearly identical speeds with respect to an ECI frame. Therefore, the local-ether model is in accord with the high synchronism among the GPS atomic clocks. Further, to keep the GPS atomic clocks synchronous with the ground clocks, the speed-dependent mass-variation factor together with the gravitational redshift has been treated by purposely adjusting the atomic clock rate before the launch of satellites [13,16].

Quantitatively, the radius r_H of the half-synchronous orbits is about 26,600 km. Hence, the various clocks move virtually at an identical speed of $v_a = \sqrt{GM/r_H}$ with respect to an ECI frame. Meanwhile, the clock stationary on the ground has the speed v_E due to earth's rotation. This speed is much lower than that of GPS satellites. Thus the mass-variation effect tends to slow down the atomic clock rate after launch. On the contrary, as the satellite is launched to the orbit of a lower gravitational potential, the gravitational effect tends to speed up the clock rate. By taking both the speed- and the gravitation-dependent effects into account, the fractional shift in transition frequency after launch to a half-synchronous orbit is given by the local-ether model as

$$\frac{\triangle f}{f_0} = \left(\frac{v_E^2}{2c^2} - \frac{v_a^2}{2c^2}\right) + \frac{MG}{c^2}\left(\frac{1}{r_L} - \frac{1}{r_H}\right), \tag{28}$$

where r_L and r_H denote the geocentric distances of GPS satellite before and after its launch, respectively.

By taking $v_E = 350$ m/sec, $v_a = 3870$ m/sec $(= 1.29 \times 10^{-5}\ c)$, $MG/c^2 = 4.43$ mm, $r_L = 6380$ km, and $r_H = 26600$ km, the fractional shift is 4.45×10^{-10}, to which the mass-variation and the gravitational effects contribute -8.3×10^{-11} and 5.28×10^{-10}, respectively. The GPS microwave L1 carrier is operated at a frequency of 1575.42 MHz, which is generated by multiplying the output of the atomic clock at 10.23 MHz by a factor of 154 [17]. This carrier is then modulated by a series of digital data at a rate of 10.23 Mbit/sec, which contain the codes for positioning. In GPS the received data rate is designed to be 10.23 Mbit/sec. Thus the atomic clock rate is actually adjusted a little slower to $10.22999999545 (= 10.23 - 4.55 \times 10^{-9})$ MHz before the launch [16]. It is seen that the calculated fractional frequency shift agrees excellently with the practice of clock-rate adjustment in GPS. If the clock-rate shift is not compensated, it tends to cause a synchronism error of as large as 38 μs in one day.

It is noted that the speed- and gravitation-dependent frequency shifts derived from the local-ether wave equation are identical to those presented in [18] based on the special and the general relativity, respectively. However, the mechanism of frequency shift and its consequence are different. According to the local-ether wave equation, the frequency shifts due to the gravitational and the mass-variation effects originate from an intrinsic quantum property of matter wave bounded in atom. After the launch to the orbit, the atomic clock rate increases slightly to 10.23 MHz. Thereafter, during the entire propagation from the satellite to the ground, the carrier frequency and the data rate remain fixed at 1575.42 MHz and 10.23 Mbit/sec, respectively, when the Doppler effect is omitted for simplicity. Thus over a duration of time, the total number of bits of data received on the ground tends to be identical to that transmitted from the satellite, as expected intuitively.

5.4 Frequency shift in earthbound and interplanetary spacecraft microwave links

Consider the frequency-shift experiment conducting in a spacecraft microwave link, where a microwave is generated from a stable source onboard a spacecraft, received by a ground station, and then compared to a reference signal from another stable source at the station. As the spacecraft moves at a high speed in a vast domain, the Doppler, the mass-variation, and the gravitational effects are incorporated in the frequency shift between the received and the reference signals.

Suppose the two microwave sources at the spacecraft and the ground station are of identical rest transition frequency f_0. According to the local-ether model, the frequency shift due to the speed-dependent mass-variation effect is given by the difference between the two transition frequencies as

$$\triangle f = f_0 \left(v_E^2 - v_{sc}^2\right)/2c^2, \qquad (29)$$

where v_{sc} and v_E are the speeds of the spacecraft and the ground station with respect to their respective local-ether frames. Speed v_{sc} is referred to a geocentric or a heliocentric inertial frame, depending on the location of the spacecraft being earthbound or interplanetary, respectively. Further, if the spacecraft comes close to a planet enough, the reference frame of speed v_{sc} should switch to the local ether associated with that planet. On the other hand, speed v_E has nothing to do with the location of the spacecraft and is referred uniquely to an ECI frame, both for the earthbound and the extraterrestrial cases.

Consider the spacecraft microwave link by Vessot *et al.*, where a microwave was generated from a stable hydrogen maser at 1.420 GHz onboard a spacecraft and was received by a ground station with a second identical maser [14,19,20]. By using microwave mixers at the ground station, the frequency shift between waves can be measured with high precision. The spacecraft was launched nearly vertically to the apogee at an altitude of 10,000 km, then fell down, and finally impacted upon ocean. The frequency shift given by the preceding formula with both v_E and v_{sc} being referred to an ECI frame agrees with that presented in this earthbound spacecraft link (in conjunction with the frequency shifts due to the Doppler and the gravitational effects).

However, for an interplanetary case, the local-ether model leads to that speeds v_{sc} and v_E in (29) are referred to a heliocentric and a geocentric inertial frames, respectively. Consider the spacecraft microwave link by Krisher *et al.*, where a spacecraft ventured on an extraterrestrial trajectory with a flyby of Saturn [21] or of Venus [22]. A microwave was transmitted at about 2.3 GHz from the spacecraft and was received by earth stations, with an onboard stable quartz oscillator and a hydrogen maser on the ground.

The frequency-shift formula presented in this interplanetary spacecraft link (in conjunction with the frequency shifts due to the Doppler and the gravitational effects) looks like (29), except that both speeds v_{sc} and v_E are in a heliocentric inertial frame [21,22]. Thus there is a discrepancy in the effect of earth's orbital motion on the speed v_E of the ground station. The linear speed due to earth's orbital motion around the Sun is about 30 km/sec, which is much higher than that due to earth's rotation. Thus the frequency shift predicted from the local-ether model deviates from the calculated result made in [21,22] by a constant amount of about $\delta f = -11.5$ Hz as $f_0 \simeq 2.3$ GHz. Meanwhile, it is reported that the frequency of the onboard quartz oscillator tends to shift due to aging. This frequency shift also contains a constant term with a coefficient determined by the least-square algorithm [22]. Thus it seems not appropriate to test the predicted constant deviation from this experiment. Anyway, according to the frequency shift given in the form of (29), the reference frame of the onboard clock has been shown to depend on the location of the spacecraft. On the other hand, to determine the reference frame of the ground clock in an interplanetary microwave link provides a means to test the local-ether wave equation.

5.5 Spatial isotropy in Hughes-Drever experiment

In the Hughes-Drever experiment, it has been found that the transition frequency of an atom or an ion is quite stable hour by hour and day by day, in spite of earth's rotational and orbital motions [23]. Similar stability in phase over propagation path is also found in the fiber-link experiment [24] and the Kennedy-Thorndike experiment [11]. This hourly and daily stability in frequency or phase is known as the spatial isotropy as the orientation and position of the Earth are changing in space.

According to the local-ether model, it is evident that earth's orbital motion around the Sun or whatever is not involved in the speed v of an earthbound particle with respect to an ECI frame. Thus the quantum energy in an atom is entirely independent of the orbital motion. Further, the quantum energy can even be invariant under earth's rotation, if the speed v remains a constant during the rotation. Such a *constant-speed condition* is fulfilled by a geostationary atom, by an atom moving at a fixed velocity with respect to the ground at a substantially fixed latitude, or by an atom moving on a circular earth's satellite orbit. For an atom satisfying the constant-speed condition, the energies of quantum states and hence the transition frequency between two states are invariant under earth's rotation, whatever the dependence of quantum energy on speed. The synchronism among the various GPS clocks can be ascribed to this isotropy. The constant-speed condition has also been applied in [5] to account for the aforementioned spatial isotropy in phase.

The transition frequency in the Hughes-Drever experiment is associated with the Zeeman-split energy states due to the interaction of nuclear spin with an external magnetic field [23]. In these experiments, the atoms or ions are in gaseous or liquid state and their ground velocities are constant under earth's rotation in statistics. Therefore, the local-ether model is in accord with the Hughes-Drever experiment, whatever the dependence of nuclear magnetic interaction on mass and speed. A variety of atoms or ions have been shown to exhibit this isotropy, including lithium [25], beryllium [26], mercury [27], and neon [28]. It is noticed that in some literature this isotropy is ascribed to the local Lorentz invariance [26–28].

6 Conclusion

The matter wave of a particle is supposed to be governed by the local-ether wave equation incorporating the natural frequency and the electric scalar potential. The time derivative in the wave equation is referred specifically to the local-ether frame, which is a geocentric or a heliocentric inertial frame, depending on the location of the particle being earthbound or interplanetary, respectively. Under the ordinary case of weak potential, a wavefunction close to a spatial harmonic tends to be close to a temporal harmonic with the angular frequency combining the natural frequency and the propagation vector. Then the local-ether wave equation leads to a first-order time evolution equation in terms of the reduced wavefunction. From the evolution equation, it is found that the velocity of a particle with respect to the local-ether frame is proportional to the spatial rate of change of the wavefunction and to the inverse of the angular frequency. For a harmonic wavefunction, the particle velocity is then proportional to the propagation vector. Further, due to the natural frequency and hence the dispersion of wavefunction, the angular frequency is equal to the natural frequency times a speed-dependent factor, which is just the famous Lorentz mass-variation factor, except the reference frame of particle speed. As the natural frequency corresponds to the rest mass of a particle, the angular frequency is then the speed-dependent mass, aside from a common scaling factor. These speed-dependent angular frequency, propagation vector, and mass derived from the local-ether wave equation are in accord with the de Broglie postulates in conjunction with the Lorentz mass-variation law, except the reference frame.

Based on Galilean transformations, the time evolution equation for a particle bounded in a moving atom reduces to a form which looks like Schrödinger's equation. However, the position vector and hence the time derivative are referred to the atom frame. Moreover, the time derivative additionally incorporates the mass-variation factor. Thereby, the energies of the quantum states due to the electrical scalar potential and hence the transition frequency decrease by this factor. The atomic clock rate is then expected to decrease with the speed of the clock by the mass-variation factor, where the speed is referred specifically to the local-ether frame. When the gravitational effect is taken into account by the perturbation method, it is seen that the transition frequency and hence the atomic clock rate also decrease under the gravitational potential. Thus, according to the local-ether wave equation, the frequency shifts due to the gravitational and the mass-variation effects originate from an intrinsic quantum property of matter wave bounded in atom.

According to the local-ether model, the speeds of earthbound atoms are referred to an ECI frame. Obviously, this is in accord with the east-west directional anisotropy in the Hafele-Keating experiment, the synchronism in GPS, and with the spatial isotropy in the Hughes-Drever experiment. Quantitatively, it has been shown that the speed- and the gravitation-dependent frequency shifts are actually in accord with the clock-rate difference in the Hafele-Keating experiment and the clock-rate adjustment in GPS. Moreover, for the atomic clock onboard the spacecraft in the earthbound or the interplanetary microwave link, the local-ether model leads to that the speed in the mass-variation factor is referred to a geocentric or a heliocentric inertial frame, respectively. This switch in reference frame is in accord with the frequency-shift formulas adopted in the spacecraft-link experiments. However, for the atomic clocks at ground stations in both links, the local-ether model predicts that the speeds are referred uniquely to an ECI frame. For the interplanetary link, the prediction leads to a constant deviation from the calculated result of frequency shift reported in the literature. This constant deviation in frequency shift is yet to be verified and then provides a means to test the local-ether wave equation.

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