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A WAVE-PROPAGATION MODEL FOR GRAVITATIONAL EFFECTS ON LIGHT DEFLECTION AND RADAR ECHO TIME

C.-C. Su

Department of Electrical Engineering
National Tsinghua University
Hsinchu, Taiwan

Abstract—The deflection of a light beam passing close to the Sun and the increment in interplanetary radar echo time are known as important consequences of the general theory of relativity. In this investigation, by endowing each celestial body a unique local ether and by modifying the speed of light in a gravitational potential, a wave-propagation model is proposed to account for these phenomena in a classical way without invoking the space-time curvature.

1 Introduction

2 Gravitational Effect on Wave Propagation

3 Light Deflection

4 Increment in GPS Propagation Time

5 Increment in Interplanetary Radar Echo Time

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1. INTRODUCTION

It is known that the deflection of a light beam passing close to the Sun and the increment in interplanetary radar echo time predicted from the general theory of relativity have been demonstrated experimentally. The general relativity invokes the space-time curvature associated with the length contraction and the time dilation caused by a gravitational field [1]. In this investigation, an entirely different interpretation of these propagation phenomena is presented.

Recently, we have proposed the local-ether model of wave propagation by slightly modifying the classical ether notion [2]. According to this model, electromagnetic wave can be viewed as to propagate via a medium like the ether. However, the ether is not universal. It is supposed that the region under sufficient influence of the gravity due to the Earth, the Sun, or another celestial body forms a local ether which in turn moves with the respective body. Each individual local ether is finite in extent and may be wholly immersed in another local ether of larger extent. Thus, the local ethers may form a multiple-level hierarchy. At a given position, it is the lowest-level local ether that determines the wave propagation locally. For an electromagnetic wave propagating within a single local ether, it is proposed that as in the classical ether notion, the propagation speed with respect to the associated local ether is just the speed of light c, independent of the motion of the source and the receiver. For the earthbound propagation, the medium is the earth local ether which as well as earth's gravitational potential is stationary in a geocentric inertial frame. While, for the interplanetary propagation, the main medium is the sun local ether stationary in a heliocentric inertial frame. Thus, for the earthbound or the interplanetary propagation, the speed c is referred to a geocentric or a heliocentric inertial frame, respectively. As in the classical ether notion, the actual propagation range is the distance from the source at the instant of wave emission to the receiver at the instant of reception. However, the reference frame is attached to the associated local ether. Owing to the movement of the receiver with respect to the local ether during wave propagation, the interplanetary propagation depends both on earth's rotation and on earth's orbital motion around the Sun. While, the earthbound propagation still depends on earth's rotation but is entirely independent of earth's orbital motion. By using the endowed flexibility of the reference frame for wave propagation, this new classical model can solve the discrepancy in the effect of earth's orbital motion and has been adopted to account for a variety of propagation phenomena consistently [2], including the earthbound GPS (global positioning system) pseudorange correction, the interplanetary radar echo time, and the earthbound and the interplanetary Doppler frequency shifts. Moreover, as examined with the present accuracy, the local-ether model is still in accord with the Michelson-Morley experiment which is known to make the classical ether notion obsolete.

In this investigation, we modify the speed of light in the presence of a gravitational potential and then show that this simple propagation model is in accord with the light deflection, the increment in GPS propagation time, and the increment in interplanetary radar echo time. Thus, by taking into account the gravitational effect on wave propagation, the local-ether model can be more complete.

2. GRAVITATIONAL EFFECT ON WAVE PROPAGATION

It is supposed that the gravity which is associated with the formation of local ethers may further be associated with the speed of wave propagation. Specifically, it is postulated that under the gravitational potential of a celestial body of mass M, the electromagnetic potential or field in free space having no sources is governed by the wave equation given as

$$\nabla^2\Psi(\mathbf{r},t) - \frac{n_g^2}{c^2}\frac{\partial^2}{\partial t^2}\Psi(\mathbf{r},t) = 0, \tag{1}$$

where the *gravitational index* n_g is a function of space proposed as

$$n_g(\mathbf{r}) = 1 + 2\frac{GM}{c^2 r}, \tag{2}$$

r $(= |\mathbf{r}| > R_0)$ is the radial distance away from the center of the celestial body of radius R_0, G is the gravitational constant, and Ψ denotes the electric scalar potential Φ or a Cartesian component of the magnetic vector potential $\mathbf{A}$, electric field $\mathbf{E}$, or of magnetic field $\mathbf{B}$. Thereby, the gravitational potential due to a celestial body tends to affect the wave propagation [3]. It is seen from the wave equation that electromagnetic wave propagates at the speed of c/n_g with respect to the local-ether frame associated with the celestial body. Thus, the region under a gravitational potential may be treated classically as a dielectric medium of which the refractive index is given by n_g. Accordingly, the speed of light passing through this region decreases from c to c/n_g, where the speed is referred to a geocentric or a heliocentric inertial frame for an earthbound or an interplanetary propagation, respectively.

3. LIGHT DEFLECTION

Based on the proposed simple propagation model associated with the gravitational index, the deflection of a light beam passing near a celestial body can be accounted for by the spatial variation of the index. It is noted that closer to the surface of the body, the index becomes larger. The deflection of light is somewhat similar to the total internal reflection of a short wave from the ionosphere of which the refractive index is decreasing with increasing altitude. In what follows, we provide a quantitative analysis of the deflection under the condition of a slowly-varying index.

For a time-harmonic wave at angular frequency ω, the wave equation (1) becomes the Helmholtz equation as

$$\nabla^2\Psi(\mathbf{r}) + k_0^2 n_g^2(\mathbf{r})\Psi(\mathbf{r}) = 0, \tag{3}$$

where $k_0 = \omega/c$. If the index n_g is a slowly-varying function of space, the wave Ψ can be close to a space harmonic $e^{-j\mathbf{k}\cdot\mathbf{r}}$, where the propagation vector $\mathbf{k}$ determines the propagation direction and the rate of phase variation. Further, the magnitude of the propagation vector can be given as

$$k = k_0 n_g. \tag{4}$$

Without loss of generality, consider a plane wave of which the phase on the plane $x = x_0$ is ϕ_0. That is,

$$\Psi(x_0, y, z) = \exp(-j\phi_0). \tag{5}$$

If the phase ϕ_0 is a constant over the plane, the propagation vector has no components in this plane ($\mathbf{k} = \hat{x}k$) and the wave is propagating in the x direction. Next, consider the phase distribution over a neighboring plane ($x = x_0 + dx$) parallel to the first plane. If the index is uniform, it is known that the wave over the second plane is simply given as

$$\Psi(x_0 + dx, y, z) = \exp(-j\phi_0 - jkdx). \tag{6}$$

If the index is slowly varying, it is expected that this phase distribution holds. Then, by applying Taylor's series expansion to the index along a transverse (say, z) direction, the wave can be rewritten as

$$\begin{aligned} &\Psi(x_0 + dx, y, z) \\ = \;& \Psi(x_0, y, z)\exp\left\{-jk_0\left[n_g + \frac{dn_g}{dz}(z - z_0)\right]dx\right\} \\ = \;& \Psi(x_0, y, z)\exp\left\{-jk_0\left[n_g - \frac{dn_g}{dz}z_0\right]dx\right\}\exp(-jk_z z), \end{aligned} \tag{7}$$

where $k_z = (k_0 dx)dn_g/dz$ and the index and its derivative are taken at a suitable point $(x_0 + dx, y, z_0)$ on the second plane. It is of essence to note that the index variation along a transverse direction introduces a space harmonic in that direction. This transverse harmonic $\exp(-jk_z z)$ in turn will cause the plane wave to deflect by an angle of $\sin^{-1}(|k_z|/k) \simeq |k_z|/k$ toward the higher index. Since $k_z = 0$ on the first plane, the rate of change of the transverse component of the propagation vector is then given by

$$\frac{dk_z}{dx} = k_0 \frac{dn_g}{dz}. \tag{8}$$

When generalized to a plane wave propagating in an arbitrary direction, the preceding equation states that the deflection over a differential propagation distance dl is equal to dl times the derivative of the index in a transverse direction divided by the index. It is noted that this agrees with the transverse component of the vector form of the differential equation of the light rays derived in geometric optics using a different approach [4].

Thereafter, consider the deflection of a light beam passing near a celestial body of radius R_0. Arrange the associated local-ether frame such that the origin is at the center of the celestial body, the wave propagates in the x direction (when the deflection is absent) along the line with $y = 0$ and $z = b$, parameter b ($\geq R_0$) is the vertical distance from the center of the celestial body to the propagation path, and the index n_g varies transversely in the z direction. Thus, the deflection in the $-z$ direction over a differential propagation length dx is related to the derivative of index as

$$\frac{d\alpha}{dx} = -\frac{dn_g}{dz}, \tag{9}$$

where we have made use of the gravitational index n_g being quite close to unity. As the deflection is very small, the propagation path is mostly along the line. Thus, the total deflection angle α is given by the integration of the derivative of index over the entire propagation path as

$$\begin{aligned} \alpha &= -\int_{-\infty}^{\infty} \frac{\partial n_g(x,z)}{\partial z} dx = -\frac{2GM}{c^2} \int_{-\infty}^{\infty} \frac{\partial}{\partial z} \left(\frac{1}{r} \right) dx \\ &= \frac{2GM}{c^2} \int_{-\infty}^{\infty} \frac{b}{(b^2 + x^2)^{3/2}} dx, \end{aligned} \tag{10}$$

where we have made use of $r^2 = x^2 + z^2$ with $z = b$. A direct integration leads to that

$$\alpha = \frac{4GM}{c^2 b}. \tag{11}$$

This result is identical to that derived from the general theory of relativity [5], although no space-time curvature is invoked here. Note that the propagation path and the geometry of calculation are referred specificality to the associated local-ether frame. Without specifying the reference frame explicitly, a similar approach of the spatially-varying index has been treated as a pedagogic aid in [1].

It is seen that the deflection depends on parameter b. The minimum value of b is R_0, which occurs for a ray grazing the celestial body. The corresponding maximum deflection is then given as

$$\alpha_{\max} = \frac{4GM}{c^2 R_0}. \tag{12}$$

For a ray grazing the Sun, $\alpha_{\max} = 4.9 \times 10^{-4}$ deg. As it is well known, the deflection of optical wave passing close to the Sun was first confirmed in as early as 1917, shortly after the introduction of the general theory of relativity. Moreover, the deflection of microwave passing near the Sun has also been demonstrated [6].

4. INCREMENT IN GPS PROPAGATION TIME

Another propagation phenomenon associated with the gravitational index n_g is the increment of propagation time for a wave passing near a celestial body. The increment in propagation time per unit propagation length is given by $(n_g - 1)/c$. Thereby, the increment in the one-way propagation time can be given by the integration of $(n_g - 1)/c$ along the propagation path from the source at the instant of emission to the receiver at the instant of reception in the local-ether frame associated with the celestial body. Note that as in the classical ether notion, the reference frame and the reference instants for the transmitter and the receiver are particularly specified. Otherwise, the propagation path is undetermined.

Consider the earthbound propagation from a transmitter onboard a GPS satellite to a receiver on the ground. Arrange the geocentric inertial frame such that the wave propagates along the x direction. Thus, the increment in the one-way propagation time due to the gravitational potential of the Earth is given by the integration over the entire propagation path as

$$\triangle\tau_g = \frac{1}{c}\int_{-x_s}^{-x_e} (n_g - 1)\, dx = \frac{2GM}{c^3}\int_{-x_s}^{-x_e} \frac{dx}{\sqrt{b^2 + x^2}}, \tag{13}$$

where $-x_s$ is the x coordinate of the transmitter at the instant of emission, $-x_e$ is that of the receiver at the instant of reception ($x_s > x_e > 0$), b $(= \sqrt{r^2 - x^2})$ is the vertical distance from earth's center to the propagation path in the geocentric frame, and M is the mass of the Earth. A direct integration leads to that

$$\triangle\tau_g = \frac{2GM}{c^3}\ln\frac{r_e - x_e}{r_s - x_s}, \tag{14}$$

where r_s $(= \sqrt{b^2 + x_s^2})$ is the geocentric distance of the transmitter at the instant of emission and r_e $(= \sqrt{b^2 + x_e^2})$ is that of the receiver at the instant of reception. The increment can be rewritten as

$$\triangle\tau_g = \frac{2GM}{c^3}\ln\frac{r_s + x_s}{r_e + x_e} = \frac{2GM}{c^3}\ln\frac{r_s + r_e + R}{r_s + r_e - R}, \tag{15}$$

where $R = x_s - x_e$ is the propagation range in a geocentric frame. The increment (14) is exactly identical to that given in [7] derived based on the general relativity, including the reference frame and the reference instants.

It is seen that the increment $\triangle\tau_g$ depends on parameter b. In GPS, $b \leq r_e \simeq R_0 \simeq 6400\,\text{km}$ and $r_s \simeq 26600\,\text{km}$ for the half-synchronous satellites. The largest and the smallest increments occur when $b = R_0$ and $b = 0$, respectively. Which in turn correspond to $R = \sqrt{r_s^2 - R_0^2}$ and $R = r_s - R_0$, respectively. Thus, the largest and the smallest increments are $\triangle\tau_g = 60$ and $43\,\text{ps}$, respectively. This one-way increment has been proposed as a subtle GPS pseudorange correction due to earth's gravitational effect [7].

5. INCREMENT IN INTERPLANETARY RADAR ECHO TIME

Finally, we consider the interplanetary propagation time, for which the associated local-ether frame changes to a heliocentric inertial frame. Particularly, we examine the increment in the delay time for a radar echo from a planet, where a microwave is transmitted from an earth-based antenna, passes near the Sun, reaches and reflected back from the target planet, passes near the Sun again, and then received by the earth-based antenna. By following the procedure in deriving (15), the increment in the round-trip propagation time due to the gravitational potential of the Sun becomes

$$\triangle\tau_g = \frac{2GM}{c^3}\left\{\ln\frac{r_{Ef} + r_P + R_f}{r_{Ef} + r_P - R_f} + \ln\frac{r_{Eb} + r_P + R_b}{r_{Eb} + r_P - R_b}\right\}, \tag{16}$$

where r_{Ef} and r_{Eb} are the heliocentric distance of the earth-based transceiver at the instants of emission and reception, respectively, r_P

is the heliocentric distance of the target planet-based reflector at the instant of reflection, R_f and R_b are respectively the forward and the backward propagation ranges in a heliocentric frame, and M is the mass of the Sun.

The fractional difference between r_{Ef} and r_{Eb} and that between R_f and R_b are of the order of normalized speed v_E/c, where v_E is earth's speed in a heliocentric frame [8]. These differences are small and can be neglected in calculating the subtle gravitational effect. If the positions of the transceiver and the reflector are referred to an appropriate instant (say, the instant of emission), then the increment in the radar echo time becomes

$$\triangle\tau_g = \frac{4GM}{c^3} \ln \frac{r_E + r_P + R_{EP}}{r_E + r_P - R_{EP}}, \tag{17}$$

where r_E and r_P are the heliocentric distances of the transceiver and the reflector, respectively, and R_{EP} is the separation distance from the transceiver to the reflector, all at the reference instant. Without clearly specifying the reference instants, the preceding echo-time increment $\triangle\tau_g$ has been demonstrated by sending microwave to Venus, Mercury, Mars, and spacecrafts passing near the Sun [9–11].

It is seen that the increment depends on parameter b ($\geq R_0$). The maximum value of the increment $\triangle\tau_g$ occurs at superior conjunction for which the ray is grazing the celestial body and $b = R_0$. In interplanetary radar, radius $R_0 \ll r_E, r_P$. Thus, $r_E + r_P = \sqrt{x_E^2 + R_0^2} + \sqrt{x_P^2 + R_0^2} \simeq R_{EP} + R_0^2(1/2x_E + 1/2x_P)$, where x_E and x_P are the projections of r_E and r_P on the propagation path, respectively. Thereby, the maximum increment in radar echo time can be approximated as

$$\triangle\tau_{g_{\max}} \simeq \frac{4GM}{c^3} \ln \frac{4r_E r_P}{R_0^2}. \tag{18}$$

For the earth-Venus radar, this maximum increment is as large as about 230 μs. This result disagrees with that given in [5] by an amount of $4GM/c^3$, which is 19.5 μs. It appears that the various deviations based on the general relativity given in [7, 9, 5] are not entirely identical.

6. CONCLUSION

By endowing each celestial body a unique local ether and by modifying the speed of light in a gravitational potential, the new classical model is used to account for the propagation phenomena known as important consequences of the general relativity, without invoking the space-time curvature. It is found that the deflection of light passing near a celestial body can be treated in a classical way as the deflection due to spatial variation of dielectric index. The derived deflection agrees with that predicted from the general relativity. The index variation leads in a consistent way to the increments in GPS propagation time and in interplanetary radar echo time. The calculated increments agree with those derived based on the general relativity. However, a fundamental difference is that in an earthbound or an interplanetary propagation, the propagation path is referred to a geocentric or a heliocentric inertial frame, respectively. Moreover, the positions of transmitter and receiver are referred to the instants of wave emission and reception, respectively. Although they are small, these differences provide a means to test the proposed wave-propagation model.

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Ching-Chuan Su was born in 1955 in Taiwan. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from National Taiwan University in 1978, 1980, and 1985, respectively. From 1980 to 1982, he was employed at the Industrial Technology Research Institute, Hsinchu, Taiwan, where he was responsible for the development of several IC fabrication processes. In 1985 he joined the faculty of National Tsinghua University, Hsinchu, Taiwan, where he is a professor in Electrical Engineering. His research areas include electromagnetic theory, quantum electromagnetic theory, numerical solutions in antenna, scattering, microstrip, waveguide, resonator, and in MOS circuit, and fabrication of ferroelectric memory IC.