

A LOCAL-ETHER WAVE EQUATION UNIFYING GRAVITATIONAL AND ELECTROMAGNETIC FORCES

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Abstract—Recently, we have presented the local-ether model of propagation of electromagnetic wave. This new classical model accounts for a wide variety of propagation phenomena, including the gravitational effects associated with the deflection of light by the Sun and the increment in interplanetary radar echo time, which are commonly ascribed to the general relativity. In this investigation, based on the local-ether model, a wave equation incorporating the gravitational potential, the electric scalar potential, and a natural frequency is proposed for the matter wave associated with a charged particle. The local-ether wave equation leads to a time evolution equation similar to Schrödinger's equation. Then both the gravitational and the electrostatic forces are derived in a quantum-mechanical approach. Furthermore, it is found that the gravitational mass and the inertial mass are identical to the natural frequency, aside from a common scaling factor. Thus the identity of gravitational and inertial mass is also derived from the local-ether wave equation.

1 Introduction

2 Local-Ether Wave Equation for Matter Wave

3 Time Evolution Equation and Force Law

4 Conclusion

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1. INTRODUCTION

Recently, we have presented the local-ether model of wave propagation [1]. That is, electromagnetic wave can be viewed as to propagate via

a medium like the ether. However, the ether is not universal. It is supposed that in the region under sufficient influence of the gravity due to the Earth, the Sun, or another celestial body, there forms a local ether which in turn moves with the gravitational potential of the respective body. For earthbound wave, the medium is the earth local ether which is stationary in a geocentric inertial frame, while the sun local ether for interplanetary wave is stationary in a heliocentric inertial frame. Consequently, an earthbound wave phenomenon can depend on earth's rotation but is entirely independent of earth's orbital motion around the Sun or whatever; while an interplanetary wave phenomenon depends on the orbital motion around the Sun as well as on the rotation. This local-ether model has been adopted to account for a wide variety of propagation phenomena, particularly the Sagnac correction in GPS (global positioning system), the time comparison by intercontinental microwave link, and the echo time of interplanetary radar [1].

Mathematically, electromagnetic wave is supposed to be governed by a wave equation based on the local-ether propagation model. Under the influence of a gravitational potential Φ_g , it is postulated that electromagnetic wave is governed by the local-ether wave equation proposed to be [2]

$$\left\{ \frac{1}{n_g} \nabla^2 - \frac{n_g}{c^2} \frac{\partial^2}{\partial t^2} \right\} \Psi(\mathbf{r}, t) = 0, \quad (1)$$

where the position vector \mathbf{r} and the time derivative are referred specifically to the associated local-ether frame and the *gravitational index* n_g is given by

$$n_g(\mathbf{r}) = 1 + \frac{2}{c^2} \Phi_g(\mathbf{r}) = 1 + 2 \frac{GM}{c^2 r}, \quad (2)$$

where G is the gravitational constant and r ($= |\mathbf{r}|$) is the radial distance away from the center of the celestial body of mass M . From the wave equation it is seen immediately that the gravitational potential tends to reduce the propagation speed of electromagnetic wave from c to c/n_g . Then the increment in propagation time per unit propagation range is given by $(n_g - 1)/c$. By integrating this term along the path from the source at the instant of wave emission to the receiver at the instant of reception with respect to a heliocentric inertial frame, it has been shown quantitatively [2] that this new classical model is in accord with the increment in interplanetary radar echo time as a microwave passing near the Sun [3, 4]. Furthermore, it has been shown quantitatively [2] that the spatial variation of the gravitational index tends to cause a

deflection of a light beam or a microwave passing near the Sun [5]. This situation is similar to the total internal reflection of a short wave from the ionosphere of which the refractive index is varying with altitude. Thus the local-ether model is in accord with these gravitational effects commonly ascribed to the general relativity.

In this investigation, based on the local-ether wave equation for electromagnetic wave, we propose a wave equation for matter wave under the influence of the gravitational and the electric scalar potentials. From this wave equation, a first-order time evolution equation similar to Schrödinger's equation can be derived for a harmonic-like matter wave. By following the procedure in deriving Ehrenfest's theorem in quantum mechanics, the velocity and then the acceleration of a charged particle under the influence of potentials can be derived. Then we explore the quantum origins of the gravity and the gravitational mass, along with those of the electromagnetic force and the inertial mass. Thereby, based on the local-ether wave equation incorporating the gravitational and the electric scalar potentials, a unified quantum theory of gravitational and electromagnetic forces is presented.

2. LOCAL-ETHER WAVE EQUATION FOR MATTER WAVE

It is supposed that matter wave associated with a particle follows the local-ether model. Under the influence of the gravitational potential Φ_g and the electric scalar potential Φ , it is postulated that the wavefunction Ψ of a particle of charge q is governed by the local-ether wave equation proposed to be

$$\left\{ \frac{1}{n_g} \nabla^2 - \frac{n_g}{c^2} \frac{\partial^2}{\partial t^2} \right\} \Psi(\mathbf{r}, t) = \frac{\omega_0^2}{c^2} \left\{ 1 + \frac{2}{\hbar\omega_0} q\Phi(\mathbf{r}, t) \right\} \Psi(\mathbf{r}, t), \quad (3)$$

where the natural frequency ω_0 as well as the charge q is an inherent constant of the effector particle, \hbar is Planck's constant divided by 2π , and the position vector \mathbf{r} and the time derivative are referred to the associated local-ether frame. For a zero natural frequency, the local-ether wave equation reduces to (1) for electromagnetic wave. In the absence of the gravitational and the electric scalar potentials, the local-ether wave equation reduces to a form which looks like the Klein-Gordan equation [6], except the reference frame. Further, when the spatial variation of the wavefunction ceases, it behaves as the harmonic $e^{-i\omega_0 t}$.

Consider the case where the gravitational potential is weak such that its value divided by c^2 is much less than unity and hence the

gravitational index n_g is quite close to unity. Thus the wave equation can be simplified. Dividing both sides of (3) by the gravitational index n_g , the local-ether wave equation for an effector particle of natural frequency ω_0 and charge q can be written as

$$\left\{ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} \Psi(\mathbf{r}, t) = \frac{\omega_0^2}{c^2} \left\{ 1 + 2 \frac{q\Phi}{\hbar\omega_0} - 2 \frac{\Phi_g}{c^2} \right\} \Psi(\mathbf{r}, t), \quad (4)$$

where the high-order terms with the gravitational potential connected to the Laplacian ∇^2 and to the scalar potential Φ are neglected.

3. TIME EVOLUTION EQUATION AND FORCE LAW

Consider the case where the potential Φ and the spatial rate of variation of the wavefunction as well as the potential Φ_g are weak. Thus the temporal variation of Ψ is close to that of the harmonic $e^{-i\omega_0 t}$. The wavefunction can be given as $\Psi(\mathbf{r}, t) = \psi(\mathbf{r}, t)e^{-i\omega_0 t}$, where the reduced wavefunction ψ has a weak temporal variation. Then its second time derivative becomes

$$\frac{\partial^2}{\partial t^2} \Psi(\mathbf{r}, t) = \left\{ \frac{\partial^2}{\partial t^2} \psi(\mathbf{r}, t) - i2\omega_0 \frac{\partial}{\partial t} \psi(\mathbf{r}, t) - \omega_0^2 \psi(\mathbf{r}, t) \right\} e^{-i\omega_0 t}. \quad (5)$$

As the temporal variation of ψ is relatively weak, the second derivative $\partial^2 \psi / \partial t^2$ can be neglected. By so doing, the wave equation (4) can be further approximated to the first-order time evolution equation in terms of the reduced wavefunction ψ as

$$i \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = -\frac{c^2}{2\omega_0} \nabla^2 \psi(\mathbf{r}, t) + \frac{1}{\hbar} q \Phi \psi(\mathbf{r}, t) - \frac{\omega_0}{c^2} \Phi_g \psi(\mathbf{r}, t), \quad (6)$$

where the two major terms $(\omega_0/c)^2 \psi$ cancel out.

As in deriving Ehrenfest's theorem in quantum mechanics, the velocity \mathbf{v} of an effector particle is supposed to be given by the time derivative of the expectation value of the position vector \mathbf{r} as

$$\mathbf{v} = \frac{d\langle \mathbf{r} \rangle}{dt}, \quad (7)$$

where the position vector \mathbf{r} and hence the velocity \mathbf{v} are referred to the local-ether frame and the expectation value of an operator O is evaluated in terms of the reduced wavefunction ψ as $\langle O \rangle = \int \psi^* O \psi d\mathbf{r}$. By expanding the local-ether-frame time derivative of the expectation

value of position vector according to the evolution equation (6), it is straightforward to show that the effector velocity is given by

$$\mathbf{v} = \frac{-ic^2}{\omega_0} \langle \nabla \rangle. \quad (8)$$

In the derivation we have made use of that the Laplacian operator is self-adjoint whereby $\int (\nabla^2 \psi_a)^* \psi_b d\mathbf{r} = \int \psi_a^* \nabla^2 \psi_b d\mathbf{r}$ and that $\nabla^2(\mathbf{r}\psi_a) = \mathbf{r}\nabla^2\psi_a + 2\nabla\psi_a$, where ψ_a and ψ_b are two arbitrary functions. It is seen that the effector velocity with respect to the local-ether frame is proportional to the expectation value of the del operator and thus is associated with the spatial rate of variation of the wavefunction. Thus the evolution equation (6) is valid only for a low-speed particle of which the matter wave has a weak spatial variation.

Further, the acceleration of the effector particle can be given in terms of the time derivative of this expectation value as

$$\mathbf{a} = \frac{-ic^2}{\omega_0} \frac{d\langle \nabla \rangle}{dt}. \quad (9)$$

By expanding the local-ether-frame time derivative of the expectation value of del operator according to the evolution equation (6), it is straightforward to show that the acceleration is given by

$$\mathbf{a} = -c^2 \frac{q}{\hbar\omega_0} \nabla\Phi + \nabla\Phi_g. \quad (10)$$

Physically, this relation states that the spatial variations of potentials Φ and Φ_g contribute to the change in the spatial variation of wavefunction ψ and then to the change in the effector velocity. However, the contribution due to potential Φ is inversely proportional to ω_0 , while that due to potential Φ_g is independent of ω_0 . The gravity is the acceleration due to the gravitational potential and thus is given as $\mathbf{g} = \nabla\Phi_g$. As the gravity is independent of the natural frequency, different kinds of particles then tend to have identical gravity at identical position.

Introduce the rest mass m_0 which is related to the natural frequency ω_0 in the familiar form of

$$m_0 = \frac{\hbar}{c^2} \omega_0. \quad (11)$$

Then, based on Newton's second law of motion with the rest mass m_0 of the effector being the inertial mass, the force exerted on a particle moving slowly under the electric scalar and the gravitational potentials is then given by

$$\mathbf{F} = -q\nabla\Phi + m_0\nabla\Phi_g. \quad (12)$$

It is noted that as the gravity is independent of the rest mass, the gravitational force is linearly proportional to this mass. As a consequence, the gravitational mass associated with this gravitational force is equal to the inertial mass under the influence of the electrostatic force. Thus the gravitational mass and the inertial mass of an effector particle stationary or moving slowly with respect to the local-ether frame are just the natural frequency of the associated matter wave, aside from the common scaling factor (\hbar/c^2). Moreover, it is noted that for the static case the electric scalar potential, similar to the gravitational potential, is proportional to the inverse of the separation distance between the effector and the source. Then both the electrostatic and the gravitational forces follow the inverse square law.

In terms of the rest mass m_0 defined from the natural frequency ω_0 , the time evolution equation (6) becomes

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m_0} \nabla^2 \psi(\mathbf{r}, t) + q\Phi \psi(\mathbf{r}, t) - m_0 \Phi_g \psi(\mathbf{r}, t), \quad (13)$$

where the position vector \mathbf{r} and the time derivative are referred to the local-ether frame. This evolution equation looks like Schrödinger's equation with the interaction being given by $(q\Phi - m_0\Phi_g)$, except the reference frame. A quantum-mechanical Hamiltonian incorporating a gravitational energy similar to that given in the preceding equation has been assumed in [7].

In addition to the electrostatic force, the electric induction force and the magnetic force have also been derived from the local-ether wave equation (3) with a slight modification by connecting the electric scalar potential to the augmentation operator [8, 9]. That is,

$$\left\{ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} \Psi(\mathbf{r}, t) = \frac{\omega_0^2}{c^2} \left\{ 1 + \frac{2}{\hbar\omega_0} q\Phi(\mathbf{r}, t)(1 + U) \right\} \Psi(\mathbf{r}, t), \quad (14)$$

where the gravitational potential is neglected and the augmentation operator U is derived from the Laplacian operator and is given by

$$U = \frac{1}{2c^2} \left(-i \frac{c^2}{\omega_0} \nabla - \mathbf{v}_s \right)^2, \quad (15)$$

where \mathbf{v}_s denotes the velocity of the source particle with respect to the local-ether frame. It is noted that the operator U tends to enhance the effect of the electric scalar potential. The inertial mass of a low-speed effector particle under the influence of the electromagnetic force is still the rest mass m_0 . Thereby, the local-ether wave equation

incorporating the natural frequency, the gravitational potential, the electric scalar potential, and the augmentation operator leads to a **unified quantum theory of gravitational and electromagnetic forces in conjunction with the identity of gravitational and inertial mass.**

4. CONCLUSION

Based on the local-ether propagation model, a wave equation incorporating the gravitational potential, the electric scalar potential, and a natural frequency is proposed for the matter wave associated with a charged particle. The involved time derivative is referred specifically to the local-ether frame. The local-ether wave equation leads to a first-order time evolution equation similar to Schrödinger's equation, with the interaction being given by the electric scalar and the gravitational potentials. Then, by following the quantum-mechanical procedure in deriving Ehrenfest's theorem, the time evolution equation leads to the well-known fact that the gravitational and the electrostatic forces are associated with the gradients of the gravitational and the electric scalar potentials, respectively. Thereby, the local-ether wave equation leads to a unified quantum theory of gravitational and electromagnetic forces. For an effector particle moving slowly with respect to the local-ether frame, it is found that the gravitational mass associated with the gravitational force and the inertial mass under the electromagnetic force are equal to the natural frequency of the associated matter wave, aside from a common scaling factor. Thereby, the local-ether wave equation also leads to the identity of gravitational and inertial mass and to their physical origin.

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