

# Modifications of Schrödinger's Equation Complying with the Effect of Earth's Rotation on Quantum Energy in Atoms and with the Electromagnetic Force

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**Abstract** – Recently, we have presented a local-ether wave equation incorporating a nature frequency and the electric scalar potential, from which the speed-dependences in the angular frequency and wavelength of matter wave, in the mass of particle, and in the energy of quantum states are derived. These relations look like the postulates of de Broglie and the Lorentz mass-variation law, except that the particle speed is referred specifically to a geocentric inertial frame and hence incorporates earth's rotation for earthbound particles. Further, the wave equation is extended by connecting the scalar potential to the augmentation operator which is associated with a velocity difference between involved particles. Then the electromagnetic force law is derived, which under some ordinary conditions reduces to the modified Lorentz force law. In this investigation, the interaction of atoms with electromagnetic radiation is explored. Then it is shown that the time evolution equation derived from the wave equation is substantially identical to Schrödinger's equation incorporating the vector potential, if the latter is observed in the atom frame and if the source generating the vector potential is electrically neutralized, as in common practice.

## 1. Introduction

Recently, we have presented a wave equation which incorporates a natural frequency  $\omega_0$  and the electric scalar potential and is proposed to govern the matter wave associated with a charged particle [1]. For a harmonic-like wavefunction, the wave equation leads to a first-order time evolution equation similar to Schrödinger's equation. Then it has been found that the energies of quantum states in an atom decrease with the atom speed by the famous Lorentz speed-dependent mass-variation factor. The propagation of matter wave as well as electromagnetic wave is supposed to follow the local-ether model [2]. Accordingly, the position vectors, time derivatives, and velocities in this wave equation are all referred specifically to an ECI (earth-centered inertial) frame for earthbound phenomena. Thus the atom speed incorporates the linear velocity due to earth's rotation. Consequently, the quantum state energy and the transition frequency of an earthbound atom depend on earth's rotation, but are entirely independent of earth's orbital motion around the Sun or others. This effect of earth's rotation on atomic quantum properties has been used to account for the east-west directional anisotropy in the atomic clock rate which in turn has been demonstrated in the Hafele-Keating experiment with circumnavigation clocks. It also accounts for the synchronism and the clock-rate adjustment in GPS (global positioning system) and the spatial isotropy in the Hughes-Drever experiment [1].

Furthermore, from the local-ether evolution equation, the velocity and then the acceleration of a charged particle under the influence of the electric scalar potential have been derived. Thus the electrostatic force is derived in conjunction with the consequence that the natural frequency  $\omega_0$  is related to the rest mass  $m_0$  of the particle in the familiar form of  $m_0 = \hbar\omega_0/c^2$  [1]. Further, the local-ether wave equation is extended in such a way that the electric scalar potential is made to connect to the augmentation operator which in turn is associated with the momentum operator and the velocity of source practices. Then the electromagnetic force law is derived. Under the low-speed condition, this law reduces to the modified Lorentz force law [3].

In this investigation, we derive the effect of the augmentation operator in the interaction of atoms with electromagnetic radiation. Then the relation between the local-ether evolution equation and Schrödinger's equation incorporating the vector potential is explored.

## 2. Schrödinger's Equation with Vector Potential

To begin with, we review the famous Schrödinger's equation and its consequences. It is well known that in the presence of the scalar potential  $\Phi$  and the vector potential  $\mathbf{A}$ , Schrödinger's equation reads

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \frac{1}{2m_0} [\mathbf{p} - q\mathbf{A}(\mathbf{r}, t)]^2 \psi(\mathbf{r}, t) + q\Phi(\mathbf{r}, t) \psi(\mathbf{r}, t), \quad (1)$$

where the operator  $\mathbf{p} = -i\hbar\nabla$ . The incorporation of the vector potential corresponds to the understanding in classical mechanics that the mechanical momentum in the Hamiltonian is given by the canonical momentum  $\mathbf{p}$  minus  $q\mathbf{A}$ .

It is known that in the presence of an external magnetic field, an individual spectral line of atoms will split into a set of closely spaced lines, known as the Zeeman splitting. The Zeeman effect has been accounted for by using quantum mechanics. For this familiar case of interaction of a hydrogen-like atom with the electromagnetic radiation from an external source, Schrödinger's equation becomes [4, 5]

$$i\hbar \frac{\partial}{\partial t} \psi = \frac{1}{2m_0} \mathbf{p}^2 \psi + q\Phi_a \psi - \frac{q}{m_0} \mathbf{A} \cdot \mathbf{p} \psi + \frac{1}{2m_0} (q\mathbf{A})^2 \psi, \quad (2)$$

where  $\Phi_a$  is the electric scalar (Coulomb) potential due to the nucleus of the atom itself and  $\mathbf{A}$  is the vector potential due to the external source. As in the literature, we have made use of  $\mathbf{p} \cdot (\mathbf{A}\psi) = \mathbf{A} \cdot \mathbf{p}\psi$  since  $\mathbf{p} \cdot \mathbf{A}$  is taken to be zero. Physically, the Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$  implies that the source generating the vector potential is electrically neutralized, as it is ordinarily. If the term  $\mathbf{p} \cdot \mathbf{A}$  does not vanish, one has  $\mathbf{p} \cdot \mathbf{A}/m_0 = (i\hbar/m_0c^2) \partial\Phi/\partial t$  from the Lorenz gauge, where potential  $\Phi$  is due to the charge associated with the non-neutralized current generating potential  $\mathbf{A}$ . The angular frequency of an ordinary time-harmonic potential is much lower than  $m_0c^2/\hbar$ . Therefore, the term  $\mathbf{p} \cdot \mathbf{A}/m_0$  is much weaker than the corresponding potential  $\Phi$  and hence is not expected to have an appreciable physical consequence even if it does exist. In our understanding, the reference frame of the position vector in wavefunction and potentials and that of the time derivative are not specified in Schrödinger's equation.

Moreover, from the generalized Ehrenfest's theorem, the electromagnetic force exerted on an unbounded charged particle due to the scalar and vector potentials can be given in terms of the expectation values as [4]

$$\mathbf{F} = m_0 \frac{d^2 \langle \mathbf{r} \rangle}{dt^2} = -q \langle \nabla \Phi \rangle - q \left\langle \frac{\partial \mathbf{A}}{\partial t} \right\rangle + \frac{1}{2m_0} q \{ \langle (\mathbf{p} - q\mathbf{A}) \times \mathbf{B} \rangle - \langle \mathbf{B} \times (\mathbf{p} - q\mathbf{A}) \rangle \}, \quad (3)$$

where the magnetic flux density  $\mathbf{B} = \nabla \times \mathbf{A}$ . As noted in [4], in this formula the operator  $\mathbf{p}$  in  $(\mathbf{p} - q\mathbf{A})$  does not commute with field  $\mathbf{B}$ . Thus the preceding force formula does not agree exactly with the Lorentz force law. The discrepancy between these two force formulas is expected to be  $(i\hbar q/2m_0) \nabla \times \mathbf{B}$ . Although this discrepancy may be small in magnitude, it has not yet solved to our knowledge.

## 3. Local-Ether Wave Equation and its Consequences

We then go on to consider the local-ether wave equation. It is postulated that under the influence of the electric scalar potential  $\Phi$ , the matter wave  $\Psi$  of a charged particle is governed by the nonhomogeneous wave equation proposed to be [1]

$$\left\{ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} \Psi(\mathbf{r}, t) = \frac{\omega_0^2}{c^2} \left\{ 1 + \frac{2}{\hbar\omega_0} q\Phi(\mathbf{r}, t) \right\} \Psi(\mathbf{r}, t), \quad (4)$$

where the natural frequency  $\omega_0$  as well as the charge  $q$  is an inherent constant of the particle, and the position vector  $\mathbf{r}$  and the time derivative are referred to the associated local-ether frame, which is an ECI frame for earthbound particles.

If the potential  $\Phi$  is weak, the wavefunction  $\Psi$  tends to be close to a space-time harmonic  $e^{i\mathbf{k}\cdot\mathbf{r}}e^{-i\omega t}$ , where  $\omega^2 = \omega_0^2 + c^2k^2$ . Thus it has been shown that the velocity of the particle is given by  $\mathbf{v} = \mathbf{k}c^2/\omega$  and hence the angular frequency can be given by  $\omega = \omega_0/\sqrt{1 - v^2/c^2}$ , where the velocity  $\mathbf{v}$  is referred specifically to the local-ether frame. As the natural frequency  $\omega_0$  is shown to be related to the rest mass  $m_0$ , the speed-dependent mass  $m$  related to the frequency  $\omega$  is then given by the familiar form of  $m = m_0/\sqrt{1 - v^2/c^2}$  [1]. It is noted that these relations of the speed-dependent angular frequency and wavelength of matter wave and of the speed-dependent mass of particle look like the postulates of de Broglie and the Lorentz mass-variation law, except that the particle speed  $v$  is referred specifically to the local-ether frame. Thereafter, by introducing the reduced wavefunction  $\psi$  given by  $\Psi(\mathbf{r}, t) = \psi(\mathbf{r}, t)e^{i\mathbf{k}\cdot\mathbf{r}}e^{-i\omega t}$  and by expanding the Laplacian and the time derivative in the d'Alembertian operator, it has been shown that the preceding wave equation in  $\Psi$  reduces to the first-order time evolution equation in  $\psi$  [1]. That is,

$$i\hbar\frac{\omega}{\omega_0}\frac{\partial}{\partial t}\psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m_0}\nabla^2\psi(\mathbf{r}, t) + q\Phi(\mathbf{r}, t)\psi(\mathbf{r}, t) - i\frac{\hbar^2}{m_0}\mathbf{k}\cdot\nabla\psi(\mathbf{r}, t). \quad (5)$$

Again, the position vector  $\mathbf{r}$  here is referred to the local-ether frame.

Consider a hydrogen-like atom which is moving at a velocity  $\mathbf{v}_a$  with respect to the local-ether frame. It is expected that the electric scalar potential  $\Phi_a$  due to the nucleus of the atom will move with this atom. Accordingly, this potential is stationary in the atom frame with respect to which the atom is stationary, while it is moving in the local-ether frame. Under Galilean transformations, the potential comoving with the atom can be written as  $\Phi_a(\mathbf{r})$  or as  $\Phi_a(\mathbf{r} - \mathbf{v}_a t)$ , where the position vector  $\mathbf{r}$  is referred to the atom or to the local-ether frame, respectively. The average value of the velocity of the electron bounded in the atom should be identical to the atom velocity  $\mathbf{v}_a$ ; otherwise, the electron tends to escape from the atom. Thus the spatial and temporal variation of the wavefunction of the bounded electron can be expected to be close to the factored-out harmonic  $e^{i\mathbf{k}\cdot\mathbf{r}}e^{-i\omega t}$  and then the reduced wavefunction is governed by the preceding evolution equation, where the propagation vector  $\mathbf{k} = m\mathbf{v}_a/\hbar$ , the speed-dependent mass  $m = m_0/\sqrt{1 - v_a^2/c^2}$ , the potential is given by  $\Phi_a(\mathbf{r} - \mathbf{v}_a t)$ , and the position vector  $\mathbf{r}$  is referred to the local-ether frame.

Remark the Galilean transformation

$$\left(\frac{\partial f}{\partial t}\right)_a = \frac{\partial f}{\partial t} + \mathbf{v}_a \cdot \nabla f, \quad (6)$$

where  $\partial/\partial t$  and  $(\partial/\partial t)_a$  denote the time derivatives with respect to the local-ether and the atom frames and are taken under constant  $\mathbf{r}$  and  $(\mathbf{r} - \mathbf{v}_a t)$ , respectively, as  $\mathbf{r}$  is referred to the local-ether frame. Thereby, for the electron bounded in the moving atom, the time evolution equation observed in the atom frame becomes [1]

$$i\hbar\frac{\omega}{\omega_0}\frac{\partial}{\partial t}\psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m_0}\nabla^2\psi(\mathbf{r}, t) + q\Phi_a(\mathbf{r})\psi(\mathbf{r}, t), \quad (7)$$

where the position vector  $\mathbf{r}$  and hence the time derivative are referred to the atom frame, instead of the local-ether one. It is noted that the time derivative connects with an extra multiplying term of  $\omega/\omega_0$  which is just the mass-variation factor  $1/\sqrt{1 - v_a^2/c^2}$ . However, except for this factor, the time evolution equation as well as the potential is independent of the motion of atom if the atom frame is adopted as the reference frame. Consequently, the solutions for the eigenfunction  $\psi$  and the eigenvalue  $\hbar\tilde{\omega}(\omega/\omega_0)$  of this equation in the atom frame will be independent of the atom speed  $v_a$ . Accordingly, as compared to that of a stationary atom, the energy  $\hbar\tilde{\omega}$  of each quantum state will decrease with the inverse of the

mass-variation factor when the atom is moving at speed  $v_a$  with respect to the local-ether frame.

The frequency of light emitted from or absorbed by an atom is known to be equal to the transition frequency which in turn is proportional to the difference in energy between two involved quantum states. Thus the transition frequency  $f$  is determined by the energy  $\hbar\tilde{\omega}$  and then decreases with increasing atom speed by the inverse of the mass-variation factor. That is,

$$f = f_0 \sqrt{1 - v_a^2/c^2}, \quad (8)$$

where the atom speed  $v_a$  is referred specifically to the local-ether frame and  $f_0$  is the rest transition frequency of the atom when it is stationary in this frame. Consequently, the transition frequency and hence the clock rate of earthbound atomic clocks depend on earth's rotation, but are entirely independent of earth's orbital motion. Thereby, the atomic clock flying westward tends to have a lower speed and tick at a faster rate than the one flying eastward. Thus the preceding formula accounts for the east-west directional anisotropy in atomic clock rate demonstrated in the Hafele-Keating experiment with circumnavigation clocks. On the other hand, for a geostationary atom or an atom onboard an earth's satellite moving in a circular orbit, the speed  $v_a$  and hence the transition frequency  $f$  remain unchanged with the passage of time. Thus the preceding formula also accounts for the high synchronism among the various GPS atomic clocks moving in nearly circular orbits and for the spatial isotropy in transition frequency in the Hughes-Drever experiment with geostationary atoms [1].

Moreover, in order to derive the whole electromagnetic force, it is proposed that the wave equation is modified by connecting the potential  $\Phi$  to a dimensionless operator  $U$ . For the electric scalar potential  $\Phi$  due to source particles of a given velocity  $\mathbf{v}_s$  with respect to the local-ether frame, it is postulated that the local-ether wave equation incorporates the operator  $U$  [3]. That is,

$$\left\{ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} \Psi(\mathbf{r}, t) = \frac{\omega_0^2}{c^2} \left\{ 1 + \frac{2}{\hbar\omega_0} q\Phi(\mathbf{r}, t)(1 + U) \right\} \Psi(\mathbf{r}, t), \quad (9)$$

where operator  $U$  is derived from the Laplacian operator and is given by

$$U = \frac{1}{2c^2} \left( -i \frac{c^2}{\omega_0} \nabla - \mathbf{v}_s \right)^2. \quad (10)$$

The operator  $U$  tends to enhance the effect of the electric scalar potential and hence is called the augmentation operator. Again, the local-ether wave equation can lead to a first-order time evolution equation in terms of the reduced wavefunction  $\psi$ . When the particle speed is low and then the propagation vector  $\mathbf{k}$  in the factored-out harmonic is taken to zero, the evolution equation reads

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \frac{1}{2m_0} \mathbf{p}^2 \psi(\mathbf{r}, t) + q\Phi(\mathbf{r}, t)(1 + U)\psi(\mathbf{r}, t), \quad (11)$$

where the position vector  $\mathbf{r}$  is referred to the local-ether frame. By evaluating the velocity and then the acceleration of the charged particle under the influence of the electric scalar potential connected to the augmentation operator in a quantum-mechanically approach, the electromagnetic force exerted on the particle has been derived [3].

Consider the force law for the ordinary case where the source particles are drifting in a matrix and the ions which constitute the matrix tend to electrically neutralize the mobile particles, such as electrons in a metal wire. Suppose the neutralizing matrix is of an arbitrary charge density  $\rho_m$  and moves as a whole at a velocity  $\mathbf{v}_m$  with respect to the local-ether frame, while the mobile source particles are of charge density  $\rho_v$  and move at  $\mathbf{v}_s$  with respect to this frame. Under the ordinary low-speed condition where all the involved particles move slowly with respect to the local-ether frame and the sources drift very slowly with respect to

the matrix frame, the electromagnetic force law for a particle of charge  $q$  and inertial mass  $m_0$  can be given in terms of the local-ether potentials  $\Phi$  and  $\mathbf{A}$  [3]. That is,

$$\mathbf{F}(\mathbf{r}, t) = q \left\{ -\nabla\Phi(\mathbf{r}, t) - \left( \frac{\partial}{\partial t} \mathbf{A}(\mathbf{r}, t) \right)_m + \mathbf{v}_{em} \times \nabla \times \mathbf{A}(\mathbf{r}, t) \right\}, \quad (12)$$

where the time derivative  $(\partial/\partial t)_m$  is referred to the matrix frame, the velocity difference  $\mathbf{v}_{em} = \mathbf{v}_e - \mathbf{v}_m$ , and  $\mathbf{v}_e$  is the velocity of the charged particle with respect to the local-ether frame. The electric scalar potential  $\Phi$  and the magnetic vector potential  $\mathbf{A}$  in turn are given by

$$\Phi(\mathbf{r}, t) = \frac{1}{\epsilon_0} \int \frac{\rho_n(\mathbf{r}', t - R/c)}{4\pi R} dv' \quad (13)$$

and

$$\mathbf{A}(\mathbf{r}, t) = \frac{1}{\epsilon_0 c^2} \int \frac{\mathbf{J}_n(\mathbf{r}', t - R/c)}{4\pi R} dv', \quad (14)$$

where potential  $\Phi$  is due to the net charge density  $\rho_n = \rho_v + \rho_m$ , potential  $\mathbf{A}$  is due to the neutralized current density  $\mathbf{J}_n = \mathbf{v}_{sm}\rho_v$ ,  $\mathbf{v}_{sm}$  ( $= \mathbf{v}_s - \mathbf{v}_m$ ) is the Newtonian relative velocity of the source particle with respect to the matrix, and  $R = |\mathbf{r} - \mathbf{r}'|$ . It has been pointed out that the formula (12) is identical to the Lorentz force law, if the latter is observed in the matrix frame, as done tacitly in common practice [3].

#### 4. Modifications of Schrödinger's Equation

In what follows, we derive from the local-ether wave equation the evolution equation for the interaction of atoms with electromagnetic radiation. Consider an atom which is moving at a velocity  $\mathbf{v}_a$  with respect to the local-ether frame. Again, the spatial variation of the wavefunction  $\Psi$  of the bounded electron is expected to be close to the space harmonic  $e^{i\mathbf{k}\cdot\mathbf{r}}$ , where the position vector  $\mathbf{r}$  is referred to the local-ether frame. Thus the Laplacian becomes

$$\nabla^2 \Psi(\mathbf{r}, t) = \left\{ \nabla^2 \tilde{\psi}(\mathbf{r}, t) + i2\mathbf{k} \cdot \nabla \tilde{\psi}(\mathbf{r}, t) - k^2 \tilde{\psi}(\mathbf{r}, t) \right\} e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (15)$$

where  $\Psi(\mathbf{r}, t) = \tilde{\psi}(\mathbf{r}, t)e^{i\mathbf{k}\cdot\mathbf{r}}$  and  $\tilde{\psi}$  is a weak function of space. The term associated with  $k^2$  is neglected hereafter, as the atom speed  $v_a$  is supposed to be much lower than  $c$ . Thereby, the local-ether wave equation (9) becomes

$$\left\{ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} \tilde{\psi}(\mathbf{r}, t) = \frac{\omega_0^2}{c^2} \tilde{\psi}(\mathbf{r}, t) + \frac{2\omega_0}{\hbar c^2} q\Phi(1 + U_k) \tilde{\psi}(\mathbf{r}, t) - i2\mathbf{k} \cdot \nabla \tilde{\psi}(\mathbf{r}, t), \quad (16)$$

where the augmentation operator  $U_k$  is defined as

$$U_k = \frac{1}{2c^2} \left( \frac{\mathbf{p}}{m_0} + \frac{\hbar\mathbf{k}}{m_0} - \mathbf{v}_s \right)^2. \quad (17)$$

It is seen that the incorporation of  $\mathbf{k}$  is owing to the manipulation that the space harmonic  $e^{i\mathbf{k}\cdot\mathbf{r}}$  is factored out from  $\Psi$ .

Consider the ordinary case where the scalar potential  $\Phi$  as well as the spatial rate of variation of  $\Psi$  is weak. Thus the temporal variation of  $\Psi$  or  $\tilde{\psi}$  is close to that of the harmonic  $e^{-i\omega_0 t}$  and then the wavefunction can be given as  $\tilde{\psi}(\mathbf{r}, t) = \psi(\mathbf{r}, t)e^{-i\omega_0 t}$ , where  $\psi$  is a weak function of space and time. Then its second time derivative becomes

$$\frac{\partial^2}{\partial t^2} \tilde{\psi}(\mathbf{r}, t) = \left\{ \frac{\partial^2}{\partial t^2} \psi(\mathbf{r}, t) - i2\omega_0 \frac{\partial}{\partial t} \psi(\mathbf{r}, t) - \omega_0^2 \psi(\mathbf{r}, t) \right\} e^{-i\omega_0 t}. \quad (18)$$

As the temporal variation of  $\psi$  is relatively weak, the second derivative  $\partial^2\psi/\partial t^2$  can be neglected. Then we have the first-order time evolution equation in terms of the reduced wavefunction  $\psi$

$$\frac{\partial}{\partial t} \psi(\mathbf{r}, t) = i \frac{c^2}{2\omega_0} \nabla^2 \psi(\mathbf{r}, t) - i \frac{1}{\hbar} q\Phi(1 + U_k) \psi(\mathbf{r}, t) - \frac{c^2}{\omega_0} \mathbf{k} \cdot \nabla \psi(\mathbf{r}, t). \quad (19)$$

We then go on to rearrange the evolution equation to express it in the atom frame, instead of the local-ether frame. The last term in the preceding equation can be written as  $-\mathbf{v}_a \cdot \nabla \psi$ , since the propagation vector can be given by  $\mathbf{k} = m_0 \mathbf{v}_a / \hbar$ . Thus, by using the Galilean transformation (6) again, the time evolution equation for the electron bounded in a moving atom becomes

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m_0} \nabla^2 \psi(\mathbf{r}, t) + q\Phi(\mathbf{r}, t)(1 + U_k)\psi(\mathbf{r}, t), \quad (20)$$

where the position vector  $\mathbf{r}$  and hence the time derivative are referred to the atom frame. As the potential in the preceding equation is given simply by  $\Phi(\mathbf{r}, t)$  with  $\mathbf{r}$  being referred to the atom frame, it has to be given by  $\Phi(\mathbf{r} - \mathbf{v}_a t, t)$  in (19) with  $\mathbf{r}$  being referred to the local-ether frame.

Ordinarily, the electromagnetic radiation in the interaction of atoms comes from a neutralized source. The electric scalar potential is supposed to be composed as  $\Phi = \Phi_a + \Phi_s + \Phi_m$ , where  $\Phi_a$  is due to the nucleus of the atom,  $\Phi_s$  to the mobile charged particles forming the current in an external source, and  $\Phi_m$  to the matrix in this neutralized source. Under complete neutralization,  $\Phi_s = -\Phi_m$ . Further, suppose the source is stationary in the atom frame, that is,  $\mathbf{v}_m = \mathbf{v}_a$ . Thus the Doppler frequency shift is not involved. Then it is easy to show that

$$(\Phi_s + \Phi_m)(1 + U_k) = \frac{\Phi_s}{2c^2} \left\{ \left( \frac{\mathbf{p}}{m_0} - \mathbf{v}_{sm} \right)^2 - \left( \frac{\mathbf{p}}{m_0} \right)^2 \right\} = -\frac{1}{m_0} \mathbf{A} \cdot \mathbf{p} + \frac{1}{2} \mathbf{A} \cdot \mathbf{v}_{sm}, \quad (21)$$

where the vector potential due to the neutralized source is given according to (14) as

$$\mathbf{A} = \frac{1}{c^2} \mathbf{v}_{sm} \Phi_s. \quad (22)$$

Thus the atom-frame evolution equation becomes

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \frac{1}{2m_0} \mathbf{p}^2 \psi(\mathbf{r}, t) + q\Phi_a(\mathbf{r})\psi(\mathbf{r}, t) - \frac{q}{m_0} \mathbf{A} \cdot \mathbf{p} \psi + \frac{1}{2} q \mathbf{A} \cdot \mathbf{v}_{sm} \psi, \quad (23)$$

where the augmentation operator connected to potential  $\Phi_a$  is neglected as the effect of its influence on the quantum states is small in the interaction. Again, the potential  $\Phi_a$  is supposed to move with the atom and is stationary in the atom frame, while it is moving in the local-ether frame. It is noted that the time evolution equation as well as the potential is independent of the motion of atom if the atom frame is adopted as the reference frame.

The preceding evolution equation then looks like Schrödinger's equation (2), except the last term and the reference frame. The last term in the preceding equation is of the second order of normalized speed  $v_{sm}/c$  and is very weak as the drift speed  $v_{sm}$  is very low, while the corresponding one in Schrödinger's equation is a quadratic term of  $q\mathbf{A}$ . It is seen that the latter is smaller in magnitude than the former by a factor of  $|q\Phi_s|/m_0 c^2$  which in turn is much less than unity. Anyway, this second-order interaction is commonly ignored in analysis [4, 5] and no quantitative measurements are reported, to our knowledge. However, one fundamental difference is that the position vector, the time derivative, and the drift velocity are referred specifically to the atom frame. In the perturbational treatment of interaction, the quantum states of the unperturbed system are usually taken from the solutions of Schrödinger's equation with a *stationary* potential  $\Phi_a$ . By so doing, one has actually adopted the atom frame as the reference frame tacitly, although the result can be frame-independent. Thereby, **the preceding evolution equation is identical to Schrödinger's equation, if the latter is observed in the atom frame** as done tacitly in common practice. In other words, Schrödinger's equation (2) with a stationary potential  $\Phi_a$  has some hidden restrictions. That is, the reference frame is actually the atom frame, the atom speed is low in the local-ether frame, the source generating the interacting potential

is electrically neutralized and is stationary in the atom frame, and the drift speed in the source is very low in this frame. However, these conditions are so common as to be ignored easily.

The evolution equation (11), (20), or (23) then presents modifications of Schrödinger equation, referred specifically either to the local-ether or to the atom frame. It is noted that the atom-frame evolution equation (23) is independent of the atom velocity itself. Therefore, its consequences comply with Galilean relativity and hence are independent of earth's motions. However, this is owing to the approximation that the  $k^2$  term in (15) is omitted. When this second-order term is retained and hence the restriction of low atom speed is removed, the effect of earth's rotation resumes as in the consequences of (7).

## 5. Conclusion

Based on the local-ether wave equation incorporating a nature frequency, the electric scalar potential, and the augmentation operator, the first-order time evolution equation is derived for a harmonic-like wavefunction. Then the effect of the augmentation operator in the interaction of atoms with electromagnetic radiation is discussed. Except a small second-order term, this evolution equation can be identical to Schrödinger's equation with the vector potential, if the latter is observed in the atom frame. In common practice, this frame has been adopted tacitly as the reference frame, as the scalar potential due to the atom is taken to be stationary. The predicted second-order interaction is stronger than the one in Schrödinger's equation and might provide a means to test the modified equation.

Besides, the local-ether wave equation leads to the speed-dependences in the angular frequency and wavelength of matter wave, in the mass of particle, and in the energy of quantum states. Thus it provides the physical origin of the postulates of de Broglie and the Lorentz mass-variation law. Moreover, it accounts for the east-west directional anisotropy in the Hafele-Keating experiment, the synchronism in GPS, and for the spatial isotropy in the Hughes-Drever experiment. It also leads to the electromagnetic force law in conjunction with the physical origin of the inertial mass. Thus the local-ether wave equation and the modified Schrödinger equation account for a variety of phenomena in a consistent way.

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