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REINTERPRETATION OF FIZEAU'S EXPERIMENT WITH MOVING MEDIUM IN ACCORD WITH THE SAGNAC EFFECT DUE TO EARTH'S ROTATION

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Abstract—The famous Fizeau's interferometry experiment with flowing water is commonly cited as a demonstration of the velocity transformation in the special relativity. In this investigation, by taking into account the modification of the propagation velocity due to the motion of dielectric medium and the modification of the propagation length due to the Sagnac effect, an entirely different interpretation of this experiment is presented. Physically, the influence of the medium velocity on the phase velocity is associated with an effect of the polarization current. Both the medium velocity and the Sagnac effect depend on earth's rotation, while its influence on the phase difference in Fizeau's experiment cancels out substantially.

1. INTRODUCTION

From the famous Fizeau's interferometry experiment with water flowing at a speed v, it is commonly inferred that the phase speed of light traveling along a moving medium of refractive index n is given by $c/n + v(1 - 1/n^2)$. An old interpretation of this modification of phase speed by the medium speed is known as the Fresnel drag effect, whereby the ether is dragged partially by the moving medium to an extent depending on the index n [1, 2]. Presently, a widely accepted explanation of this modification is ascribed to the special relativity, whereby the velocity addition of c/n and v based on the Lorentz transformation leads to the aforementioned phase speed [1, 2].

Recently, we have presented the local-ether model of propagation of electromagnetic wave [3]. It is supposed that electromagnetic wave propagates via a medium like the ether. However, the ether is neither universal nor dragged by a moving medium. It is supposed that in the region under a sufficient influence of the gravity due to the Earth, the Sun, or another celestial body, there forms a local ether which as well as the associated gravitational potential moves with the respective body. Thereby, for earthbound or interplanetary waves, the propagation is referred uniquely to a geocentric or a heliocentric inertial frame, respectively. Thus earth's rotation contributes to the Sagnac effect which in turn is associated with the modification of the propagation length and time due to the movement of the propagation path during wave propagation. For the interplanetary propagation, earth's orbital motion contributes to the Sagnac effect as well. This local-ether model has been adopted to account for the Sagnac effect due to earth's motions in a wide variety of propagation phenomena, particularly the global positioning system (GPS), the intercontinental microwave link, and the interplanetary radar [3]. As examined within the present accuracy, the local-ether model is still in accord with the Michelson-Morley experiment which is known to make the classical ether notion obsolete. Moreover, by modifying the speed of light in a gravitational potential, this simple propagation model leads to the deflection of light by the Sun and the increment in the interplanetary radar echo time which are important phenomena supporting the general theory of relativity [4].

In this investigation, an entirely different interpretation of Fizeau's experiment is presented. Based on the local-ether wave equation, the phase velocity of a plane wave propagating in a moving dielectric medium is derived. Further, the Sagnac effect due to the movement of the optical path with earth's rotation is also taken into account to analyze the phase difference in the interferometry. The calculated phase velocity and phase difference are then compared with those of the special relativity and of the interference experiment. The reference frames of the propagation velocity and of the medium velocity are particularly examined.

2. LOCAL-ETHER WAVE EQUATION OF ELECTRIC FIELD

Consider the ordinary case with the conduction current where the ensemble of mobile charged particles which form the current are flowing in a matrix, such as electrons in a metal wire. Ordinarily, the ions which constitute the matrix tend to electrically neutralize the mobile particles. Thus the net charge density ρ_n is much less in magnitude than ρ_v, the charge density of the mobile particles. If the net charge density vanishes or its contribution is negligible, the current density which participates in electromagnetic phenomena is then given by the *neutralized current density* [5]

$$\mathbf{J}_n(\mathbf{r}, t) = (\mathbf{v}_s - \mathbf{v}_m)\rho_v(\mathbf{r}, t), \tag{1}$$

where $\mathbf{v}_s$ and $\mathbf{v}_m$ are the velocities of the mobile particles and the neutralizing ions, respectively. It is noted that the velocity difference represents the drift velocity of the mobile particles with respect to the neutralizing matrix, regardless of the frame of the involved particle velocities.

It is known that the electric scalar and the magnetic vector potentials associated with ρ_n and $\mathbf{J}_n$ are governed by the wave equations

$$\nabla^2\Phi(\mathbf{r}, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi(\mathbf{r}, t) = -\frac{1}{\epsilon_0}\rho_n(\mathbf{r}, t) \tag{2}$$

and

$$\nabla^2\mathbf{A}(\mathbf{r}, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A}(\mathbf{r}, t) = -\frac{1}{\epsilon_0 c^2}\mathbf{J}_n(\mathbf{r}, t). \tag{3}$$

Based on the local-ether model of wave propagation presented in [3], the position vector $\mathbf{r}$ and the time derivative $\partial/\partial t$ in these wave equations are referred specifically to an earth-centered inertial (ECI) frame for earthbound experiments. Thereby, the propagation speed of the potentials is the constant c with respect to this frame.

Further, it has been shown in [5] that the electromagnetic phenomena associated with the Lorentz force law can be accounted for, if electric field is given in terms of the potentials by

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\Phi(\mathbf{r}, t) - \left(\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r}, t)\right)_m, \tag{4}$$

where $(\partial/\partial t)_m$ denotes the time derivative referred to the matrix frame with respect to which the neutralizing matrix is stationary. The matrix-frame time derivative of the potential states that electric field and the electric induction force are due to the time rate of change in the potential experienced by the affected particle which in turn is stationary in the matrix frame. In the scattering with a dielectric medium, this time derivative reflects the situation that the polarization charges interacting with each other are substantially stationary in this frame. Thereby, the local-ether wave equation of electric field is

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r}, t) = \frac{1}{\epsilon_0}\nabla \rho_n(\mathbf{r}, t) + \frac{1}{\epsilon_0 c^2}\left(\frac{\partial}{\partial t}\mathbf{J}_n(\mathbf{r}, t)\right)_m. \quad (5)$$

It is noted that in this equation the time derivative of the neutralized current density is referred to the matrix frame, which happens to be the one of the drift velocity associated with this current density.

Consider the case with a dielectric medium of permittivity ϵ, where $\mathbf{J}_n = (\partial \mathbf{P}/\partial t)_m$ for the polarization current and the polarization vector $\mathbf{P} = (\epsilon - \epsilon_0)\mathbf{E}$. The polarization vector is due to the displacement of the polarization charge relative to the ions of the matrix under the influence of electric field. The polarization current is thus associated with the time derivative of the polarization vector with respect to the matrix frame, just as the drift velocity of the mobile particles forming a conduction current is referred to this frame. For a uniform plane wave propagating in a uniform medium, the polarization $\mathbf{P}$ has no variation in the polarization direction. Thus the polarization charge vanishes, as seen from the continuity equation. Thereby, the wave equation of electric field becomes

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = \frac{\epsilon - \epsilon_0}{\epsilon_0}\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2}\right)_m. \quad (6)$$

It is noted that the time derivative of electric field associated with the polarization current is referred to the matrix frame, different from the one associated with the d'Alembert operator.

3. PHASE SPEED OF PLANE WAVE IN MOVING DIELECTRIC MEDIUM

Consider the simpler case where a z-polarized uniform plane wave propagates along the x direction in a uniform dielectric medium which in turn moves at a velocity $\mathbf{v}_m$ with respect to an ECI frame. That is, $\mathbf{E}(\mathbf{r}, t) = \hat{z}E_0 e^{ikx}e^{-i\omega t}$, where the coordinate x is referred to an ECI frame, ω is the angular frequency, and E_0 is an arbitrary constant.

Thus, to the first order of matrix speed, the wave equation (6) reduces to an algebraic equation in the propagation constant k

$$k^2 + 2(n^2 - 1)\frac{v_{mx}\omega}{c^2}k - n^2\frac{\omega^2}{c^2} = 0, \tag{7}$$

where $v_{mx} = \mathbf{v}_m \cdot \hat{x}$, the refractive index $n = \sqrt{\epsilon/\epsilon_0}$, and we have made use of the Galilean transformation $(\partial^2 f/\partial t^2)_m = \partial^2 f/\partial t^2 + 2v_{mx}\partial^2 f/\partial x \partial t$ for an arbitrary function f. Then it is easy to show that the propagation constant is given by $k = k_0 \{n + (1 - n^2)v_{mx}/c\}$, where $k_0 = \omega/c$. It is noted that this propagation constant depends on the speed v_{mx}.

The phase speed c_m $(= \omega/k)$ of the uniform plane wave propagating in the moving uniform dielectric medium is then given by

$$c_m = \frac{c}{n} + v_{mx}\left(1 - \frac{1}{n^2}\right). \tag{8}$$

Physically, the dependence of the phase speed on the medium speed as well as the familiar dependence on the refractive index is associated with the effect of the polarization current. This phase speed is similar to the speed obtained from the velocity transformation in the special relativity. However, aside from the physical origin, a fundamental difference is that the phase speed c_m and the matrix velocity $\mathbf{v}_m$ are referred specifically to an ECI frame. Thus the matrix velocity $\mathbf{v}_m$ incorporates the linear velocity due to earth's rotation, even for a geostationary medium. Furthermore, it is noted that the preceding formula is applicable only to a uniform plane wave propagating in a uniform dielectric medium and is correct only to the first order of matrix speed. Thus the applicability of the phase-speed formula has some hidden restrictions. This presents another discrepancy from the special relativity.

4. SAGNAC EFFECT AND FIZEAU'S EXPERIMENT

Fizeau's experiment deals with the fringe shift due to interference between two light beams propagating in opposite directions along a water-flowing pipe. Suppose that the water is flowing at a velocity $\mathbf{v}_f$ with respect to the pipe which in turn moves at a velocity $\mathbf{v}_l$ with respect to an ECI frame. Then the matrix velocity $\mathbf{v}_m = \mathbf{v}_f + \mathbf{v}_l$. For an electromagnetic wave propagating in an arbitrary $\hat{l}$ direction along a linear segment of the optical path in the interferometer, the propagation constant can be written as

$$k = k_0\left\{n + \left(1 - n^2\right)\hat{l} \cdot \mathbf{v}_m/c\right\}, \tag{9}$$

where n and $\mathbf{v}_m$ are of the dielectric medium filling the path segment.

Further, owing to the movement of the path during wave propagation, the propagation range which represents the actual propagation length is not simply equal to the path length l. As in the classical model of wave propagation, the propagation range l_p depends on the velocity $\mathbf{v}_l$ of the path segment with respect to an ECI frame. The modification in propagation due to the difference between the propagation range l_p and the path length l is known as the Sagnac effect, which has been demonstrated in a wide variety of propagation experiments, including those mentioned in the Introduction, as discussed elaborately in [3]. By following the derivation of the Sagnac effect for an electromagnetic wave propagating in free space discussed in [3], the propagation range l_p for a wave propagating in the $\hat{l}$ direction along a moving segment is given, to the first order of normalized speed, by

$$l_p = l \left\{ 1 + \hat{l} \cdot \mathbf{v}_l / (c/n) \right\}. \tag{10}$$

Thus even for a geostationary path, earth's rotation contributes to the Sagnac effect, just as to the Sagnac pseudorange correction in the high-precision GPS for a geostationary receiver [3].

Then the phase variation $d\phi$ over a pipe of directed differential length $\hat{l}dl$ can be given from the product of the two preceding formulas. To the first order of normalized speed, we have

$$d\phi = k_0 dl \left\{ n + (1 - n^2)\hat{l} \cdot \mathbf{v}_f / c + \hat{l} \cdot \mathbf{v}_l / c \right\}. \tag{11}$$

It is seen that the phase variation depends on the path velocity $\mathbf{v}_l$. However, the effect of this velocity on phase variation is difficult to detect in Fizeau's experiment. This is because that the optical path for interference is closed, part of the path is filled with flowing water and part is merely with air [2]. It is noted that in the preceding formula the velocity $\mathbf{v}_l$ does not connect to the index n. As the path velocity $\mathbf{v}_l$ is substantially uniform over the interferometer, its contribution to the phase variation cancels out collectively over a closed path, regardless of the actual structure of the path.

If the minute variation in the path velocity due to the one in the geocentric position is taken into account, the last term in the preceding formula leads to another consequence of the Sagnac effect demonstrated in the Michelson-Gale experiment. In that experiment the closed optical path is formed by a beam splitter and a series of mirrors, which is similar to the one in Fizeau's experiment, except for the flowing water. It is known that the corresponding phase variation is proportional both to the rate of earth's rotation and to the area enclosed by the path. However, this phase variation is difficult to measure, unless the loop area is as large as a fraction of a square kilometer as the one adopted by Michelson and Gale in their successful detection of earth's rotation with a loop interferometer in 1925 [3]. By the way, it is seen that the results of the Michelson-Gale experiment or the Sagnac rotating-loop experiment remain unchanged if the air filling the optical path is replaced by some dielectric medium comoving with the path. This consequence is in accord with Harzer's experiment discussed in [6] and with the practice adopted in fiber gyroscopes [7].

In Fizeau's experiment the two beams to be interfered propagate in opposite directions in each individual segment of the path. Thus $\hat{l}\cdot\mathbf{v}_f = \pm v_f$. It is seen from (11) that those parts of the path filled with air or with a dielectric at rest in the pipe do not contribute to the phase difference between the beams. The contribution comes only from the water-flowing path. Suppose that the water speed v_f is uniform and the total length of the water-flowing path is l. Then the difference in phase variation between the two counterpropagating beams is given by

$$\Delta\phi = 2k_0 l(n^2 - 1)v_f/c. \qquad (12)$$

It is noted that the phase difference is linearly proportional to $(n^2 - 1)$ and to the water speed v_f with respect to the pipe. The preceding phase-difference formula agrees with the interference fringe shift observed in Fizeau's experiment. It is noted that, as the Sagnac effect has been taken into account, the derivation given here is consistent with the wave propagation in GPS, the Michelson-Gale experiment, fiber gyroscopes, and in various other experiments discussed elaborately in [8].

5. CONCLUSION

Based on the local-ether model of wave propagation and electromagnetic force, a wave equation of electric field is presented, where the time derivative in the d'Alembert operator is referred specifically to an ECI frame for earthbound experiments. Meanwhile, the time derivative of electric field associated with the polarization current is referred specifically to the matrix frame, as a consequence of the situation that the drift speed associated with the neutralized current density and the time derivative relating the vector potential to electric field are referred to the matrix frame. Thereafter, the phase speed of a uniform plane wave propagating in a moving dielectric medium is derived. It is seen that this phase speed is similar to the speed obtained from the velocity transformation in the special relativity. However, the discrepancy is that the medium velocity as well as the phase speed itself is referred to an ECI frame and hence incorporates the linear velocity due to earth's rotation, aside from some restrictions in our derivation.

By taking into account both the modification of the propagation constant due to the motion of dielectric medium and the modification of the propagation length due to the Sagnac effect, the difference in phase variation between the two counterpropagating beams in Fizeau's experiment is derived. It is seen that the phase difference depends on the water speed with respect to the pipe, as the effects of earth's rotation substantially cancel out. Thus the local-ether wave equation is in accord with Fizeau's interference experiment to the first order of normalized speed. However, the roles of earth's rotation, the polarization, and of the Sagnac effect make the results somewhat different from those based on the special relativity. This may provide a means to test the local-ether wave equation, especially when the measurement precision can be raised to the second order or when the loop area can be enlarged to a fraction of a square kilometer.

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