Local-Ether Wave Equation of Electric Field and Interferometry Experiments with Moving Medium and Path

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Abstract – Recently, we have presented the local-ether model, whereby the propagation of earthbound waves is supposed to be referred uniquely to a geostationary inertial frame. Further, in order to comply with this propagation model, the modified Lorentz force law is developed. Thereby, the corresponding wave equations of potentials and fields are derived in this investigation. It is shown that the local-ether wave equation of electric field can account for various precision interferometry experiments in a consistent way, including the one-way-link experiment with a geostationary fiber, the Sagnac rotating-loop experiment with a comoving or a geostationary dielectric medium, and Fizeau's experiment with a moving dielectric medium in a geostationary interferometer. These experiments together then provide a support for the local-ether wave equation. Meanwhile, some other phenomena are predicted, which provide a means to test its validity.

1. Introduction

Recently, we have presented the local-ether model of wave propagation whereby electromagnetic wave is supposed to propagate via a medium like the ether [1]. However, the ether is not universal. Specifically, it is proposed that in the region under sufficient influence of the gravity due to the Earth, the Sun, or another celestial body, there forms a local ether which in turn moves with the gravitational potential of the respective body. Thereby, each local ether together with the gravitational potential moves with the associated celestial body. Thus, as well as earth's gravitational potential, the earth local ether for earthbound propagation is stationary in an ECI (earth-centered inertial) frame. Consequently, earthbound wave phenomena can depend on earth's rotation but are entirely independent of earth's orbital motion around the Sun or whatever. Meanwhile, the sun local ether for interplanetary propagation is stationary in a heliocentric inertial frame. This local-ether model has been adopted to account for the effects of earth's motions in a wide variety of propagation phenomena, particularly the Sagnac correction in GPS (global positioning system), the Sagnac effect in rotating-loop interferometers, the time comparison via intercontinental microwave link, and the echo time in interplanetary radar. As examined within the present accuracy, the local-ether model is still in accord with the Michelson-Morley experiment which is known to make the classical ether notion obsolete. Moreover, by modifying the speed of light in a gravitational potential, this simple propagation model leads to the deflection of light by the Sun and the increment of echo time in the interplanetary radar which are important phenomena supporting the general theory of relativity [1].

Further, it is noticed that the electric current generating the magnetic field is commonly electrically neutralized. That is, the mobile charged particles which form the current are actually drifting in a matrix and the ions which constitute the matrix tend to electrically neutralize the mobile particles, such as electrons in a metal wire. Thus the velocity which determines the current density is the drift velocity of the mobile particles with respect to the neutralizing matrix. Consequently, the neutralized current density remains unchanged when observed in different reference frames. In order to comply with the local-ether propagation model and the frame-independence of the current density, we have developed an electromagnetic force law which complies with Galilean transformations while it can reduce to a familiar form under some common condition [2]. In this new classical theory all of the involved position vectors, time derivatives, and velocities are referred specifically to their respective reference frames. Owing to this simple feature, the associated potentials and electromagnetic force remain unchanged when observed in different frames. Under the common low-speed condition where the source particles forming the current drift very slowly with respect to the neutralizing matrix, this force law reduces to a form similar to the Lorentz force law. However, the fundamental modification is that the current density generating the magnetic field, the time derivative applied to the magnetic vector potential in the electric induction force, and the particle velocity connecting to the magnetic field in the magnetic force are all referred specifically to the matrix frame in which the matrix is stationary. The modified equation is identical to the Lorentz force law, if the latter is observed in the matrix frame as done tacitly in common practice.

The propagation of the electromagnetic potentials is supposed to follow the local-ether model. Accordingly, the wave equations of potentials are derived in this investigation. Further, based on the modified Lorentz force law and the associated definitions of electric and magnetic fields in terms of the potentials, the local-ether wave equations of fields are derived. Thereby, the phase speed and propagation constant of the electromagnetic wave propagating in a moving medium are explored. Then, after the Sagnac effect due to the movement of the propagation path is taken into consideration, we present consistent reinterpretations for some precision interferometry experiments, including the fiber-link experiment with a geostationary setup and a geostationary optical fiber, the Sagnac rotating-loop experiments with a comoving or a geostationary dielectric medium, and Fizeau's experiment with a geostationary interferometer and a moving dielectric medium.

2. Modified Lorentz Force Law

Based on Galilean transformations, we have presented an electromagnetic force law expressed in terms of the augmented scalar potential, which is the electric scalar potential enhanced slightly with a term associated with the difference between the velocities of the effector and the source particles [2]. Consider the electromagnetic force exerted on an effector particle of charge q due to an ensemble of source particles of charge density ρ_v drifting in a matrix of charge density ρ_m. The drift velocity of the mobile source particles with respect to the matrix is given by the difference $\mathbf{v}_{sm} = \mathbf{v}_s - \mathbf{v}_m$, where $\mathbf{v}_s$ and $\mathbf{v}_m$ are the velocities of the source particles and the matrix, respectively, both with respect to a particular frame. The matrix can be a metal wire, a dielectric medium, or a magnet and tends to electrically neutralize the mobile source particles to some extent.

Under the common low-speed condition where the speeds of the involved particles are low and the source particles drift very slowly with respect to the matrix, the electromagnetic force law proposed in terms of the augmented potentials has been shown to reduce to the modified Lorentz force law [2]

$$\mathbf{F}(\mathbf{r}, t) = q \left\{ \mathbf{E}(\mathbf{r}, t) + \mathbf{v}_{em} \times \mathbf{B}(\mathbf{r}, t) \right\}, \tag{1}$$

where the velocity difference $\mathbf{v}_{em}$ $(= \mathbf{v}_e - \mathbf{v}_m)$ is the velocity of the effector particle with respect to the matrix and $\mathbf{v}_e$ is the effector velocity with respect to the aforementioned particular frame. The electric and magnetic fields in turn are defined explicitly in terms of the potentials Φ and $\mathbf{A}$ as

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\Phi(\mathbf{r}, t) - \left(\frac{\partial}{\partial t} \mathbf{A}(\mathbf{r}, t) \right)_m \tag{2}$$

and

$$\mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t), \tag{3}$$

where the time derivative $(\partial/\partial t)_m$ is referred specifically to the matrix frame. Quantitatively, the electric scalar potential Φ and the magnetic vector potential $\mathbf{A}$ are defined explicitly as

$$\Phi(\mathbf{r}, t) = \frac{1}{\epsilon_0} \int \frac{\rho_n(\mathbf{r}', t - R/c)}{4\pi R} dv' \tag{4}$$

and

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{\epsilon_0 c^2}\int \frac{\mathbf{J}_n(\mathbf{r}', t - R/c)}{4\pi R} dv', \tag{5}$$

where the net charge density $\rho_n = \rho_v + \rho_m$, the neutralized current density $\mathbf{J}_n = \mathbf{v}_{sm}\rho_v$, the propagation range $R = |\mathbf{r} - \mathbf{r}'|$, and the position vectors $\mathbf{r}$ and $\mathbf{r}'$ are referred to a particular frame.

Obviously, the values of the neutralized current density and the potentials remain unchanged when observed in different frames. Moreover, since the time derivative in the definition of field $\mathbf{E}$ and the effector velocity in the force law are referred uniquely to the matrix frame, the values of fields $\mathbf{E}$ and $\mathbf{B}$ and hence the electromagnetic force exerted on a charged particle given in the preceding formulas also remain unchanged in different frames. That is, based on Galilean transformations, the values of potentials, fields, and electromagnetic force are independent of reference frame. Further, it is noted that the drift velocity $\mathbf{v}_{sm}$ involved in the neutralized current density is a Newtonian relative velocity and hence this current density complies with Galilean relativity. For quasi-static case where the propagation time R/c in the potentials can be neglected, the potentials Φ and $\mathbf{A}$ then comply also with Galilean relativity and comove with the matrix.

It is seen that the force law (1) looks like the Lorentz force law. However, the fundamental modification is that the current density generating the magnetic vector potential, the time derivative applied to this potential in the electric induction force, and the particle velocity connecting to the curl of this potential in the magnetic force are all referred specifically to the matrix frame. Meanwhile, in applying the Lorentz force law to the analysis of, say, a motor or the magnetic deflection, the adopted reference frame is usually the one with respect to which the magnetic field and the magnet are stationary. Thus the matrix frame has been adopted tacitly as the reference frame. Therefore, the modified equation is identical to the Lorentz force law, if the latter is observed in the matrix frame as done tacitly in common practice.

3. Local-Ether Wave Equations of Potentials and Fields

Based on the local-ether model of wave propagation and the electromagnetic force in terms of the potentials, it is supposed here that each individual source particle continuously excites the electric scalar potential and hence other local-ether potentials. These potentials in turn propagate radially outward from the source position at an isotropic speed c with respect to the associated local-ether frame, independent of the motions of source and effector. That is, the position vectors $\mathbf{r}$ and $\mathbf{r}'$ and hence the propagation range R in the potentials Φ and $\mathbf{A}$ defined in (4) and (5) are referred specifically to the local-ether frame and the ratio R/c represents the propagation time from the source point $\mathbf{r}'$ at the instant t' $(= t - R/c)$ of wave emission to the field point $\mathbf{r}$ at the instant t. To comply with the frame of the position vectors, the velocities $\mathbf{v}_e$, $\mathbf{v}_s$, and $\mathbf{v}_m$ are also referred to the local-ether frame.

By applying the Laplacian operator to both sides of the integral formulas for potentials (4) and (5) and then expanding the Laplacians of the time-dependent charge and current densities divided by R [3], it can be shown that the wave equations of these local-ether potentials are given by

$$\nabla^2\Phi(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi(\mathbf{r},t) = -\frac{1}{\epsilon_0}\rho_n(\mathbf{r},t) \tag{6}$$

and

$$\nabla^2\mathbf{A}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A}(\mathbf{r},t) = -\mu_0\mathbf{J}_n(\mathbf{r},t), \tag{7}$$

where $\mu_0 = 1/\epsilon_0 c^2$ and the position vector $\mathbf{r}$ together with the time derivative $\partial/\partial t$ is referred to the local-ether frame, the reference frame of the wave propagation.

Then, by manipulating the wave equations of potentials according to the definitions of fields, the wave equations of fields can be derived. By so doing, we have the local-ether wave equation of magnetic field

$$\nabla^2 \mathbf{B}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{B}(\mathbf{r},t) = -\mu_0 \nabla \times \mathbf{J}_n(\mathbf{r},t) \tag{8}$$

and the local-ether wave equation of electric field

$$\nabla^2 \mathbf{E}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t) = \frac{1}{\epsilon_0}\nabla \rho_n(\mathbf{r},t) + \mu_0 \left(\frac{\partial}{\partial t}\mathbf{J}_n(\mathbf{r},t)\right)_m , \tag{9}$$

where, again, the position vector $\mathbf{r}$ together with the time derivative $\partial/\partial t$ is referred to the local-ether frame. It is noted that in the preceding equation the time derivative applied to the current density is referred to the matrix frame and the velocity of the mobile source particles forming this current density is also referred to this frame. In free space having no sources, the local-ether wave equations take a simpler form of

$$\nabla^2 \Psi(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi(\mathbf{r},t) = 0, \tag{10}$$

where Ψ denotes the scalar potential Φ or any Cartesian component of the vector potential $\mathbf{A}$, electric field $\mathbf{E}$, or magnetic field $\mathbf{B}$. It is seen that both electric and magnetic fields as well as the potentials propagate at the speed c with respect to the local-ether frame. These local-ether wave equations made their debut in [4].

Consider the case of wave propagation in a uniform magnetic medium of permeability μ, where $\mathbf{J}_n = \nabla \times \mathbf{M}$ for the magnetization current and the magnetization vector $\mathbf{M} = (1/\mu_0 - 1/\mu)\mathbf{B}$. Thereby, the wave equation of magnetic field becomes

$$\frac{\mu_0}{\mu}\nabla^2 \mathbf{B}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{B}(\mathbf{r},t) = 0, \tag{11}$$

where we have made use of the relation $\nabla \cdot \mathbf{B} = 0$, which in turn is a consequence of (3). It is seen that in a uniform magnetic medium of μ, electromagnetic wave propagates at the speed $c\sqrt{\mu_0/\mu}$ with respect to the local-ether frame. In the preceding derivation, no information about the motion of the magnetic medium is involved. Thus the derived propagation speed is independent of the motion of a uniform magnetic medium.

Consider the analogous case of wave propagation in a dielectric medium of permittivity ϵ, where

$$\mathbf{J}_n(\mathbf{r},t) = \left(\frac{\partial}{\partial t}\mathbf{P}(\mathbf{r},t)\right)_m \tag{12}$$

for the polarization current and the polarization vector $\mathbf{P} = (\epsilon - \epsilon_0)\mathbf{E}$. Note that the polarization current density is associated with the time derivative of the polarization vector with respect to the matrix frame, since the displacement of the polarization charge under the influence of electric field is relative to the ions forming the matrix and the drift velocity $\mathbf{v}_{sm}$ of the source particles forming a neutralized current is also referred to the matrix frame. Another consequence pertinent to this drift velocity is that the conservation of charge leads to another relation between $\mathbf{J}_n$ and ρ_v of

$$\nabla \cdot \mathbf{J}_n(\mathbf{r},t) = -\left(\frac{\partial}{\partial t}\rho_v(\mathbf{r},t)\right)_m . \tag{13}$$

This relation is just the continuity equation, except that the time derivative applied to the charge density is referred specifically to the matrix frame. Further, this matrix-frame continuity equation can be given in terms of the net charge density as

$$\nabla \cdot \mathbf{J}_n(\mathbf{r},t) = -\left(\frac{\partial}{\partial t}\rho_n(\mathbf{r},t)\right)_m . \tag{14}$$

This is owing to the fact that the matrix charge density ρ_m is time-independent in the matrix frame. The preceding continuity equation leads to a relation between the polarization charge density and the polarization vector as $\rho_n = -\nabla \cdot \mathbf{P}$, where we have made use of the initial condition that a uniform polarization ($\nabla \cdot \mathbf{P} = 0$) corresponds to the complete neutralization ($\rho_n = 0$). Thereby, by using electric field to express the net charge density and the neutralized current density induced in a dielectric medium, the wave equation of electric field becomes

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = -\frac{1}{\epsilon_0} \nabla\nabla \cdot [(\epsilon - \epsilon_0)\mathbf{E}(\mathbf{r}, t)] + \frac{\epsilon - \epsilon_0}{\epsilon_0} \frac{1}{c^2} \left(\frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} \right)_m . \tag{15}$$

It is noted that this wave equation involves two time derivatives of electric field referred to different frames.

Remark that based on Galilean transformations, the time derivatives of an arbitrary function f of space and time with respect to the matrix and the local-ether frames are related by

$$\left(\frac{\partial f}{\partial t} \right)_m = \frac{\partial f}{\partial t} + \mathbf{v}_m \cdot \nabla f, \tag{16}$$

where the time derivatives $\partial/\partial t$ and $(\partial/\partial t)_m$ are understood to be taken under constant $\mathbf{r}$ and $(\mathbf{r} - \mathbf{v}_m t)$, respectively, as the position vector $\mathbf{r}$ is referred to the local-ether frame. Further, from the preceding relation, the second-order time derivative in the matrix frame can be given in terms of the derivatives in the local-ether frame as

$$\left(\frac{\partial^2 f}{\partial t^2} \right)_m = \frac{\partial^2 f}{\partial t^2} + 2 \left(\mathbf{v}_m \cdot \nabla \right) \frac{\partial f}{\partial t} + \left(\mathbf{v}_m \cdot \nabla \right) \left(\mathbf{v}_m \cdot \nabla \right) f. \tag{17}$$

The Galilean formulas (16) and (17) have been given in [5] and [6], respectively.

Then, by using the Galilean formula (17), the wave equation of electric field can be rewritten as

$$\begin{aligned} \nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{\epsilon}{\epsilon_0} \frac{1}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} &= -\frac{1}{\epsilon_0} \nabla\nabla \cdot [(\epsilon - \epsilon_0)\mathbf{E}(\mathbf{r}, t)] \\ &+ \frac{\epsilon - \epsilon_0}{\epsilon_0} \frac{1}{c^2} \left[2 \left(\mathbf{v}_m \cdot \nabla \right) \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} + \left(\mathbf{v}_m \cdot \nabla \right) \left(\mathbf{v}_m \cdot \nabla \right) \mathbf{E}(\mathbf{r}, t) \right] . \end{aligned} \tag{18}$$

It is noted that this wave equation of electric field involves the matrix velocity with respect to the local-ether frame. This feature is quite different from that in the wave equation of magnetic field (11). Physically, this difference is due to the situation that polarization current is associated with the matrix-frame time derivative of field, while magnetization current is with a space derivative. In terms of the matrix-frame time derivatives, the local-ether wave equation of electric field can be rewritten as

$$\nabla^2 \mathbf{E} - \frac{\epsilon}{\epsilon_0} \frac{1}{c^2} \left(\frac{\partial^2 \mathbf{E}}{\partial t^2} \right)_m = -\frac{1}{\epsilon_0} \nabla\nabla \cdot [(\epsilon - \epsilon_0)\mathbf{E}] - \frac{2}{c^2} \left(\mathbf{v}_m \cdot \nabla \right) \left(\frac{\partial \mathbf{E}}{\partial t} \right)_m , \tag{19}$$

where the second-order term of the normalized speed v_m/c is ignored. It is seen that the last term in the preceding matrix-frame wave equation is smaller in magnitude than the other terms by a factor of the order of the normalized speed v_m/c.

Consider the simpler case where a z-polarized uniform plane wave propagates along the x direction in a uniform dielectric medium. The medium is moving at a velocity $\mathbf{v}_f$ with respect to a laboratory frame which in turn is moving at a velocity $\mathbf{v}_0$ with respect to the local-ether frame. Based on Galilean transformations, the matrix velocity with respect to the local-ether frame is simply $\mathbf{v}_m = \mathbf{v}_f + \mathbf{v}_0$. Then, to the first order of normalized speed, the wave equation (18) can be simplified to a form of

$$\frac{\partial^2}{\partial x^2} E_z(x, t) - \frac{\epsilon}{\epsilon_0} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E_z(x, t) = 2 v_{mx} \frac{\epsilon - \epsilon_0}{\epsilon_0} \frac{1}{c^2} \frac{\partial^2}{\partial x \partial t} E_z(x, t), \tag{20}$$

where $v_{mx} = \mathbf{v}_m \cdot \hat{x}$. It is noted that the term $\nabla \cdot (\epsilon - \epsilon_0)\mathbf{E}$ associated with the polarization charge vanishes, since both permittivity ϵ and field $\mathbf{E}$ are supposed to have no variations along the z direction of the field.

Suppose that the solution of this wave equation is of a harmonic form as $E_z = E_0 e^{ikx} e^{-i\omega t}$, where x is referred to the local-ether frame and E_0 is an arbitrary constant. Then the preceding wave equation immediately leads to an algebraic equation

$$k^2 - \frac{\epsilon}{\epsilon_0}\frac{\omega^2}{c^2} = -2v_{mx}\frac{\epsilon - \epsilon_0}{\epsilon_0}\frac{1}{c^2}k\omega. \tag{21}$$

It is easy to show that to the first order of normalized speed, the propagation constant k can be given in terms of the angular frequency ω as

$$k = \frac{\omega}{c}\left\{\sqrt{\frac{\epsilon}{\epsilon_0}} - \frac{v_{mx}}{c}\frac{\epsilon - \epsilon_0}{\epsilon_0}\right\}. \tag{22}$$

It is noted that this propagation constant depends on the speed v_{mx}, in addition to the familiar dependence on permittivity ϵ. Consequently, this relation presents a modification of the propagation constant due to the motion of the dielectric medium with respect to the local ether. Physically, the dependence on the matrix speed originates from the situation that the polarization current and its time derivative in the wave equation are referred to the matrix frame rather than to the local-ether one.

The phase speed c' $(= \omega/k)$ of the electromagnetic wave propagating in the moving dielectric medium is then given by

$$c' = \frac{c}{n} + v_{mx}\left(1 - \frac{1}{n^2}\right), \tag{23}$$

where the phase speed is referred to the local-ether frame and the refractive index $n = \sqrt{\epsilon/\epsilon_0}$. It is seen that the component of the matrix velocity parallel to the propagation direction affects the phase speed, while the transverse components do not.

The wave equation (20) can also be written in the matrix frame. To the first order of normalized speed, this wave equation reads

$$\frac{\partial^2}{\partial x^2}E_z(x,t) - \frac{\epsilon}{\epsilon_0}\frac{1}{c^2}\left(\frac{\partial^2}{\partial t^2}E_z(x,t)\right)_m = -2v_{mx}\frac{1}{c^2}\left(\frac{\partial^2}{\partial x \partial t}E_z(x,t)\right)_m. \tag{24}$$

It can be shown that $k = (\omega_m/c)(n + v_{mx}/c)$ and $c'_m = \omega_m/k = c/n - v_{mx}/n^2$, where ω_m and c'_m are the angular frequency and the phase speed of the wave when observed in the matrix frame, respectively. It is seen that

$$c'_m = c' - v_{mx}, \tag{25}$$

which states that the difference in the observed phase speed between the two reference frames is just the relative speed between the frames along the propagation direction. In other words, the derived phase speeds in the two frames comply with Galilean transformations.

The phase speed given in (23) is similar to the speed obtained from the velocity transformation in the special relativity, which in turn is known to have been demonstrated in the famous Fizeau's interferometry experiment with flowing water. However, the fundamental difference is that the phase speed c' and the matrix velocity $\mathbf{v}_m$ are referred specifically to the local-ether frame. Thus the matrix velocity $\mathbf{v}_m$ incorporates the laboratory velocity $\mathbf{v}_0$ with respect to the local-ether frame. An old interpretation of the dependence of the phase speed on the medium velocity given in (23) is known as the Fresnel drag effect [6, 7], whereby the ether is dragged partially by the moving medium to an extent depending on the index n. If n is large enough, $c' \simeq c/n + v_{mx}$ and $c'_m \simeq c/n$. It is seen that the phase speed with respect to the moving medium is independent of the velocity of the medium.

Thus it seems that the ether is completely dragged by the medium. However, according to the wave equation of electric field (15), the earth local ether is still stationary in an ECI frame, regardless of the permittivity and the velocity of a terrestrial medium. The change in the phase speed of a wave propagating in a dielectric medium, stationary or moving, is due to the effect of the polarization which in turn is induced by and related to electric field. Further, it is noted that the interpretation similar to the ether dragging is applicable only for a uniform plane wave propagating in a uniform dielectric medium and is correct only to the first order. In other words, according to the local-ether wave equation, the phase-speed formula (23) does not hold in a magnetic medium and can be no longer valid in a nonuniform dielectric medium where polarization charge emerges. Thus the applicability of the phase-speed formula has some hidden restrictions. This presents a viewpoint different from the ether-dragging notion and another discrepancy from the special relativity.

4. Reexamination of Various Interferometry Experiments

In this section we reexamine some precision interferometry experiments reported in the literature to test the local-ether wave equation of electric field. They include the fiber-link experiment with a geostationary setup and a geostationary optical fiber, the Sagnac loop interferometry with a rotating path and a comoving or geostationary dielectric medium, and Fizeau's experiment with a geostationary interferometer and a moving dielectric medium. For these earthbound experiments, the local ether is stationary in an ECI frame and hence earth's rotation should be taken into consideration, even for a geostationary medium or path. On the other hand, these experiments are entirely independent of earth's orbital motion around the Sun or others.

4.1. Phase variation with moving medium and path

First of all, consider an interferometer of which each segment of propagation path has a fixed shape and is implemented with a pair of mirrors in air, with a pipe filled with flowing dielectric liquid, or with a solid dielectric fiber. Remark that for an electromagnetic wave propagating in an arbitrary direction $\hat{l}$ along a path filled with a dielectric medium moving at a velocity $\mathbf{v}_m$ with respect to the local-ether frame, the propagation constant can be written as

$$k = k_0 \left\{ n + \left(1 - n^2\right) \hat{l} \cdot \mathbf{v}_m / c \right\}, \tag{26}$$

where $k_0 = \omega/c$ and n is the refractive index of the uniform moving medium.

Further, consider a propagation path segment of length l, such as a linear section of the pipe or fiber. Owing to the movement of the segment during the wave propagation through it, the propagation range which represents the actual propagation length is not actually l. As in the classical propagation model, the propagation range l' depends on the velocity of the path segment with respect to the local ether. The influence on propagation due to the difference between the propagation range l' and the path length l is known as the Sagnac effect. By following the derivation of the Sagnac effect for electromagnetic wave propagating in free space discussed elaborately in [1], the propagation range l' for a wave propagating in the direction $\hat{l}$ along a moving segment, when given to the first order of normalized speed, is

$$l' = l \left\{ 1 + n\hat{l} \cdot \mathbf{v}_l / c \right\}, \tag{27}$$

where $\mathbf{v}_l$ is the velocity of the path segment with respect to the local-ether frame.

It is supposed that the phase variation over the propagation path is given by $\phi = kl'$. To the first order of normalized speed, the phase variation associated with the moving medium and path is then given from the two preceding formulas by

$$\phi = k_0 l \left\{ n + (1 - n^2)\hat{l} \cdot \mathbf{v}_m / c + n^2 \hat{l} \cdot \mathbf{v}_l / c \right\}. \tag{28}$$

It is seen that the phase variation depends on the longitudinal (to the propagation path) $\hat{l}$ components of the medium velocity and the path velocity both with respect to the local-ether frame. Furthermore, these longitudinal-speed terms connect with the refractive index. Thereby, in addition to the familiar effect on the propagation constant, the refractive index also has effects on the matrix-velocity modification of the propagation constant and the path-velocity modification of the propagation length. The preceding phase-variation formula can be rewritten as

$$\phi = k_0 l \left\{ n + n^2 \hat{l} \cdot \mathbf{v}_{lm}/c + \hat{l} \cdot \mathbf{v}_m/c \right\}, \tag{29}$$

where $\mathbf{v}_{lm} = \mathbf{v}_l - \mathbf{v}_m$ denotes the Newtonian relative velocity of the moving path with respect to the moving dielectric medium. It is seen that one of the velocity-dependent terms depends on the refractive index and is associated with a Newtonian relative velocity, while the other velocity-dependent term is independent of the index and is associated with the individual velocity of the dielectric medium. For the case where the medium comoves with the propagation path ($\mathbf{v}_{lm} = 0$), the velocity-dependent phase variation is no longer dependent on the index.

4.2. Geostationary fiber-link experiment

Then we consider the experiment of one-way fiber link by Krisher *et al.*, where the phase difference between two waves generated from two identical stable hydrogen masers at 100 MHz was measured by using a network analyzer [8]. The two masers are separated by a long distance of 21 km and linked by a stable optical fiber. One signal was fed directly into the analyzer, while the other was used to modulate a laser carrier to propagate over the fiber. For a geostationary fiber, $\mathbf{v}_m = \mathbf{v}_l = \mathbf{v}_E$, where $\mathbf{v}_E$ is the linear velocity due to earth's rotation. Then formula (29) immediately leads to that the phase variation $d\phi$ over a geostationary fiber segment of differential length dl is given by

$$d\phi = k_0 \left(n + \hat{l} \cdot \mathbf{v}_E/c \right) dl. \tag{30}$$

Thus the phase variation over a propagation path L of length l is given by the path integral

$$\phi = k_0 \left(nl + \frac{1}{c} \int_L \mathbf{v}_E \cdot d\mathbf{l} \right), \tag{31}$$

where the directed differential length $d\mathbf{l} = \hat{l} dl$. It is noted that the velocity-dependent part is independent of the refractive index, since the velocity difference $\mathbf{v}_{lm} = 0$. Furthermore, it is noted that although the velocity-dependent phase variation is due to earth's rotation, its value is invariant under this rotation. This is because that both $\mathbf{v}_E$ and $d\mathbf{l}$ change in a coordinated way such that their dot product remains unchanged during earth's rotation. This invariance is in accord with the spatial isotropy demonstrated in the one-way fiber-link experiment, where it is observed that the phase variation is highly stable (as determined from the phase difference between the two waves measured every a few seconds during a couple of days), regardless of earth's rotational and orbital motions [8].

When the fiber link is placed in a geographically small region such that the velocity $\mathbf{v}_E$ due to earth's rotation is substantially constant, the phase variation over the link becomes

$$\phi = k_0 \left(nl + \frac{1}{c} \mathbf{v}_E \cdot \mathbf{d} \right), \tag{32}$$

where $\mathbf{d}$ is the directed separation distance from the end through which the light enters the fiber to the other end. It is noted that the phase variation is independent of the actual wiring, but depends on the orientation of the distance $\mathbf{d}$ with respect to the ground. If the fiber-link experimental setup is put on a turntable or a rotor, it is expected that the phase variation will change as the turntable is rotating even slowly. From the minor term in the preceding formula, the dependence of the phase variation on the orientation of the rotor is given by

$$\delta\phi = \frac{\omega}{c^2} \omega_E R_E |\mathbf{d}| \cos\theta_l \cos\theta_T, \tag{33}$$

where R_E is earth's radius, θ_l is the latitude, θ_T is the angle of the distance $\mathbf{d}$ from the east, and the turntable is suitably positioned on a horizontal plane so that the gravitational effect on ω is avoided. The phase variation associated with earth's rotation is independent of the refractive index of the fiber and hence is identical to the one in the case with a free-space link discussed in [1]. It is noted that the phase variation becomes maximum and minimum, as the distance $\mathbf{d}$ points to the east and the west, respectively. Furthermore, the phase variation changes sinusoidally with the angle θ_T. This dipole anisotropy in phase variation predicted in the proposed one-way-link rotor experiment then provides a means to test the local-ether wave equation.

4.3. Sagnac rotating-loop experiments

Consider the Sagnac effect in a rotating-loop interferometer associated with the interference between two coherent waves corotating and counterrotating with respect to the propagation loop, respectively. Commonly, the propagation path is composed of beam splitter and mirrors in air. Then the phase difference between the two waves results from the situation that the modification of propagation length due to the rotation of the loop is different between these waves propagating in opposite directions. Here, we deal with a propagation path filled with a general dielectric medium rather than a free space.

Suppose that the loop is rotating about an axis as observed from a laboratory, which in turn is rotating with the Earth. The velocity of the path segment with respect to the an ECI frame is given by $\mathbf{v}_l = \mathbf{v}_I + \mathbf{v}_{E0} + \mathbf{v}_0$, where $\mathbf{v}_0$ is the laboratory velocity with respect to an ECI frame referred to a suitable point $\mathbf{r}_0$ on the interferometer rotation axis, $\mathbf{v}_I = \bar{\omega}_I \times (\mathbf{r} - \mathbf{r}_0)$ and $\mathbf{v}_{E0} = \bar{\omega}_E \times (\mathbf{r} - \mathbf{r}_0)$ denote the other contributions to the path velocity due to the rotations of the loop and the Earth at the directed rotation rates $\bar{\omega}_I$ and $\bar{\omega}_E$, respectively, and $\mathbf{r}$ is the position vector of the path segment in the frame of the reference point $\mathbf{r}_0$.

Consider the case where the medium is comoving with the rotating loop. Thus the velocity difference $\mathbf{v}_{lm} = 0$ and hence formula (29) leads to that the phase variation $d\phi$ over a propagation path of differential length dl is given by

$$d\phi = k_0 \left(n + \hat{l} \cdot \mathbf{v}_l / c \right) dl. \tag{34}$$

The unit vectors $\hat{l}$ denoting the propagation directions of the two counterpropagating waves are antiparallel to each other in each path segment. Thus the difference in the phase variation over the loop L between the two waves is then given by the path integral

$$\triangle\phi = \frac{2k_0}{c} \oint_L (\mathbf{v}_I + \mathbf{v}_{E0}) \cdot d\mathbf{l}, \tag{35}$$

where we have made use of the facts that the major term $k_0 n dl$ in (34) is identical for the two waves and hence its contributions to the phase difference cancel out and that the contributions of a constant vector $\mathbf{v}_0$ to the phase variation over a closed path cancel out collectively, regardless of the actual structure of the loop.

Suppose the loop is coplanar. Then, by using a vector identity, the phase difference can be given by

$$\triangle\phi = \frac{4k_0}{c} (\bar{\omega}_I + \bar{\omega}_E) \cdot \mathbf{S}, \tag{36}$$

where $\mathbf{S}$ $(= \frac{1}{2} \oint_L (\mathbf{r} - \mathbf{r}_0) \times d\mathbf{l})$ denotes the directed area enclosed by loop L. It is noted that the phase difference is independent of the index n and hence is identical to the one in the case with a free-space path discussed in [1]. Again, this independence is owing to the coincidence that the index-dependent modification of the propagation constant happens to cancel the one of the propagation length. This null effect of the refractive index on the phase difference in the Sagnac loop interferometry has been demonstrated experimentally by Harzer in as early as 1914 [9]. Further, the preceding phase-difference formula has been put in practical use in fiber gyroscopes [10-12]. Alternative derivations of this formula by taking the Sagnac effect and the modifications of phase speed into account can be found in [9-11]. Anyway, in spite of the restriction on reference frame, the local-ether wave equation of electric field is in accord with the Sagnac interferometry experiment with a rotating loop and a comoving dielectric medium. On the other hand, the local-ether wave equation precludes the possibility of detecting earth's orbital motion by using earthbound fiber gyroscopes.

Next, consider the case where the medium is geostationary while the propagation loop is still rotating with respect to the laboratory. Thus $\mathbf{v}_{lm} = \mathbf{v}_I$, $\mathbf{v}_m = \mathbf{v}_{E0} + \mathbf{v}_0$, and hence formula (29) leads to that the phase variation $d\phi$ over a path of differential length dl is given by

$$d\phi = k_0 dl \left(n + n^2 \hat{l} \cdot \mathbf{v}_I / c + \hat{l} \cdot \mathbf{v}_{E0}/c + \hat{l} \cdot \mathbf{v}_0 / c \right). \tag{37}$$

The corresponding phase difference between the waves becomes

$$\triangle\phi = \frac{4k_0}{c}(n^2 \bar{\omega}_I + \bar{\omega}_E) \cdot \mathbf{S}. \tag{38}$$

It is seen that the phase difference then depends on the index n which in turn connects to the loop rotation rate $\bar{\omega}_I$. Again, the phase variation associated with earth's rotation is independent of the index and the phase difference is independent of the laboratory velocity $\mathbf{v}_0$. Ordinarily, $\omega_E \ll \omega_I$ and hence the phase difference is substantially proportional to n^2. This index-dependence of the phase difference is identical to that given in [9], although the approach is quite different. The increase of phase difference in the presence of a stationary dielectric medium has been demonstrated experimentally by Dufour and Prunier in 1942 [9]. The various index-dependences among the terms connected to the rate $\bar{\omega}_I$ or $\bar{\omega}_E$ in (36) and (38) then provide another means to test the local-ether wave equation.

4.4. Fizeau's experiment with moving medium

The last case is then the one where the interferometer is stationary while the medium is moving. Consider Fizeau's experiment dealing with the interference between two optical waves propagating in opposite directions along a stationary pipe filled with flowing water. For generality, suppose that the pipe together with beam splitters, mirrors, and other components of the interferometer is stationary in a laboratory frame which in turn moves at a velocity $\mathbf{v}_0$ with respect to an ECI frame. As the interferometer is geographically small, the path velocity is substantially constant over the setup. Thus the path velocity $\mathbf{v}_l = \mathbf{v}_0$. Further, suppose that the water is flowing at a velocity $\mathbf{v}_f$ with respect to the pipe. Then the velocity difference $\mathbf{v}_{lm} = -\mathbf{v}_f$, the matrix velocity $\mathbf{v}_m = \mathbf{v}_f + \mathbf{v}_0$, and hence formula (29) leads to that the phase variation $d\phi$ over a pipe of differential length dl is given by

$$d\phi = k_0 dl \left\{ n + (1 - n^2) \hat{l} \cdot \mathbf{v}_f / c + \hat{l} \cdot \mathbf{v}_0 / c \right\}, \tag{39}$$

where n is the index of the flowing water and $\hat{l} \cdot \mathbf{v}_f = \pm v_f$ for the two beams.

It is seen that the phase variation depends on the laboratory velocity. However, the effect of this velocity on phase variation can not be detected in Fizeau's experiment. This is because that the optical path actually adopted in the experiment is closed, part of the path is filled with flowing water and part is merely with air (see the figure in [13] or [14]). It is noted that in the preceding formula the velocity $\mathbf{v}_0$ does not connect to the index n. Again, the phase variation over a closed path given by the circulation of a constant vector $\mathbf{v}_0$ is zero.

As in the Sagnac loop interferometry, the two waves to be interfered propagate in opposite directions in each individual segment of the path. From the preceding formula it is seen that those parts of the path filled with air or with a dielectric at rest with the pipe do not contribute to the phase difference between the waves. The contribution comes only from the water-flowing pipe. Suppose that the water speed v_f is uniform and the total length of the water-flowing pipe is l. Then the difference in phase variation between the two counterpropagating waves is given by

$$\triangle\phi = 2k_0 l(n^2 - 1)v_f/c. \tag{40}$$

It is noted that the phase difference is linearly proportional to $(n^2 - 1)$ and to the water speed v_f with respect to the pipe. The preceding phase-difference formula agrees with the interference fringe observed in Fizeau's experiment with various speeds v_f and indices n.

More precisely, the path velocity $\mathbf{v}_l$ is not actually a constant value $\mathbf{v}_0$ over the closed path. That is, $\mathbf{v}_l = \mathbf{v}_{E0} + \mathbf{v}_0$ and $\mathbf{v}_m = \mathbf{v}_f + \mathbf{v}_{E0} + \mathbf{v}_0$, where $\mathbf{v}_0$ is referred to a suitable point in the setup. Hence the Sagnac effect in a loop interferometer due to earth's rotation should also appear in Fizeau's experiment. However, as earth's rotation rate is relatively slow, its effect in Fizeau's experiment is ordinarily much smaller than that due to the motion of the medium, just as its effect in the rotating-loop experiment is ordinarily much smaller than that due to the rotation of the loop. Thereby, the phase difference is substantially independent of earth's rotation. Furthermore, at least to the first order of normalized speed, Fizeau's experiment together with the Sagnac rotating-loop experiments is independent of the laboratory velocity $\mathbf{v}_0$ and hence complies with Galilean relativity, in spite of the restriction on reference frame of the medium and the path velocities.

5. Conclusion

Based on the local-ether model of wave propagation, the propagation of the potentials is referred specifically to an ECI frame in earthbound experiments. Further, under the ordinary condition of low drift speed, the electromagnetic force can be given in terms of electric and magnetic fields which in turn are given explicitly in terms of the local-ether potentials. The position vectors, time derivatives, particle velocities, propagation velocity, and current density involved are all referred specifically to their respective reference frames. Consequently, the values of potentials, fields, and force remain unchanged in different frames.

Further, based on the definitions of fields in terms of potentials, the local-ether wave equations of fields are derived. From the wave equation of magnetic field, the phase speed of electromagnetic wave in a uniform magnetic medium is found to depend on the permeability, but not on the motion of the medium. However, from the wave equation of electric field, the phase speed of a uniform plane wave propagating in a moving uniform dielectric medium is found to depend on the longitudinal component of the medium velocity and hence to incorporate the familiar Fresnel drag coefficient. This phase speed looks like the speed obtained from the velocity transformation in the special relativity. However, the fundamental difference is that the phase and the matrix speeds are referred specifically to the local-ether frame. Moreover, this phase-speed formula is not expected to hold in a magnetic or a nonuniform dielectric medium.

By taking the matrix-velocity modification of the propagation constant and the Sagnac path-velocity modification of the propagation length into account, the phase variation over a moving path filled with a moving dielectric medium is presented. Thereby, this phase-variation formula is applied to analyze various precision interferometry experiments in a consistent way. It is found that this formula is actually in accord with the spatial isotropy in the one-way fiber-link experiment, with the null effect of permittivity in the Sagnac interferometer with a dielectric medium comoving with the rotating loop, with the increase of phase difference in the Sagnac interferometer with a geostationary dielectric medium, and with the dependence of phase difference on the speed and index of the flowing water in Fizeau's experiment. These together provide a support for the local-ether wave equation of electric field. Moreover, it is predicted that as the fiber-link experimental setup is put on a turntable, the phase variation over the link will change sinusoidally as the orientation of turntable is changing. This proposed one-way-link rotor experiment, the predicted null effect of earth's orbital motion in earthbound interferometers, the predicted various index-dependences among the phase-shift terms connected to the rotation rate of the loop or of the Earth in the Sagnac interferometer, and the aforementioned discrepancies in the phase-speed formula for a moving medium and the restrictions on this formula then provide different approaches to test the local-ether wave equation.

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