Reexamination of Barnett's Experiment Based on the Modified Lorentz Force Law

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Abstract – Barnett's experiment demonstrates that the induction on a stationary cylindrical capacitor in the presence of a rotating magnet or solenoid is zero. In this investigation, based on the modified Lorentz force law, which complies with Galilean transformations and depends on relative velocities, the induction on the capacitor is reexamined. When the rotating solenoid is long and the capacitor is placed inside the solenoid, it is seen that the induction actually vanishes as observed in Barnett's experiment. However, when the capacitor is placed outside the solenoid or when the solenoid is short, it is shown that the induction can departure from zero. This prediction provides a means to test the validity of the modified Lorentz force law.

1. Introduction

It is known that in as early as 1831 Faraday demonstrated the unipolar induction on a metallic wire which is rotating in the presence of a magnet, where the rotation axis is parallel to the magnetic field. Meanwhile, it seems that the induction is determined by the relative motion between the wire and the magnet. Thus it is expected that a similar induction can be observed for a stationary wire and a rotating magnet or solenoid carrying a current. In 1912 Barnett conducted experiments to justify this issue of relative motion [1]. In Barnett's experiment two coaxial conducting cylinders of different radii are placed inside a solenoid of larger radius concentrically (see Fig. 1). The two cylinders which form a capacitor are connected to an electrometer with conducting wires. The solenoid was made to rotate about its center axis and to carry a current. The induced electromotive force will drive charges of opposite signs on to the two cylinders of the capacitor via the connecting wires, respectively. By using switches which provide suitable electric insulation, the charge on the capacitor can be measured and then the induction can be determined. However, for the case where the solenoid is rotating and the capacitor together with the connecting wires is stationary, the observed charge is merely a minute fraction of the expected amount, within the limits of experimental errors [1]. Without using the switches, Kennard also tried to examine the effect of relative motion by measuring the potential difference across a similar cylindrical capacitor placed inside or outside the concentric solenoid [2, 3]. Negative results were also reported, except for the disturbances which were attributed to the induction when the magnetization was reversed [3]. Thus it seems that the principle of relativity which states that physical laws depend on relative motion is violated. On the other hand, it is argued that the principle of relativity applies for the relative motion of translation, but not for the relative motion of rotation. Discussions on the effects of relative rotational motion on the induction still remain active in the literature [4-6]. But this issue seems not yet solved conclusively with quantitative analysis.

In this investigation we present a reexamination of Barnett's experiment by resorting to the modified Lorentz force law, which is derived from a wave equation in a quantum mechanical way based on the local-ether model of wave propagation [7]. This local-ether wave

equation in turn leads to a unified quantum theory of the gravitational and electromagnetic forces in conjunction with the origin and identity of the gravitational and inertial mass. Furthermore, this wave equation accounts for a wide variety of phenomena in modern physics, including the Sagnac effect with wave propagation, Fizeau's experiment with moving media, the gravitational redshift in the Pound-Rebka experiment, and the Hafele-Keating experiment with fast-moving atomic clocks [7]. For quasi-static cases, the modified force law is not new, as it can be derived from the Riemann force law which in turn has been proposed in as early as 1861 and can reduce to the Lorentz force law under some common situations [8].

As well as the Riemann force, the modified Lorentz force law is in compliance with Galilean transformations and can be in accord with the principle of relativity as it depends on relative velocities. Qualitatively, these force laws immediately account for the difference between the relative motions of rotation and translation. For two small objects, a relative motion of rotation is equivalent to a relative motion of translation at a given instant. However, if either object is large, the situation can be different. When the aforementioned solenoid is rotating and the connecting wires along with the capacitor are stationary, the relative motions between a given segment of the wires and different segments of the solenoid are different in direction. This is different from the case where the capacitor is rotating and the solenoid is stationary, as the relative motions between a given rotating segment and various segments of the solenoid then become identical at a given instant. Based on the modified Lorentz force law, the vanishing induction with a rotating solenoid can be accounted for without the breakdown of the principle of relativity. Further, a fundamentally different consequence is pointed out. That is, the induction on the stationary capacitor can be different from zero when the capacitor is placed outside the rotating solenoid or when the solenoid is short in length. This prediction provides a means to test the validity of the Riemann force law and the modified law.

2. Modified Lorentz Force Law

It is proposed that the electromagnetic force exerted on a charged particle due to various source particles can be given in terms of the augmented scalar potential. Precisely, the augmented scalar potential $\breve{\Phi}$ due to source particles of charge density ρ_v and experienced by the effector particle is given explicitly by the integral over a volume containing all the source particles

$$\breve{\Phi}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \left(1 + \frac{v_{es}^2}{2c^2}\right) \frac{\rho_v(\mathbf{r}', t - R/c)}{R} dv', \tag{1}$$

where $v_{es} = |\mathbf{v}_{es}|$, the velocity difference $\mathbf{v}_{es} = \mathbf{v}_e - \mathbf{v}_s$, $\mathbf{v}_e$ is the velocity of the effector located at position $\mathbf{r}$ at instant t, $\mathbf{v}_s$ is that of the source particles distributed at position $\mathbf{r}'$ at an earlier instant t' $(= t - R/c)$, the propagation range R $(= |\mathbf{r} - \mathbf{r}'|)$ is the distance from the source point $\mathbf{r}'$ to the field point $\mathbf{r}$, and R/c denotes the propagation delay time from the source to the effector at the respective positions and instants.

Further, it is postulated that the electromagnetic force exerted on the effector of charge q is given in terms of the augmented scalar potential by [7]

$$\mathbf{F}(\mathbf{r}, t) = q\left\{-\nabla\breve{\Phi}(\mathbf{r}, t) + \left(\frac{\partial}{\partial t}\sum_i \hat{\imath}\frac{\partial}{\partial v_{ei}}\breve{\Phi}(\mathbf{r}, t)\right)_e\right\}, \tag{2}$$

where $v_{ei} = \mathbf{v}_e \cdot \hat{\imath}$, $\hat{\imath}$ is a unit vector, the index $i = x, y, z$, and the time derivative $(\partial/\partial t)_e$ is referred to the effector frame with respect to which the effector is stationary. Physically, an effector-frame time derivative represents the time rate of change in some quantity experienced by the effector. It is noticed that this formula resembles Lagrange's equations adopted by Weber, Riemann, and Thomson in the early development of electromagnetic force [9,

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10]. And the augmented scalar potential resembles the velocity-dependent potential energy introduced by Riemann [9].

Ordinarily, the magnetic force is due to a conduction current where the mobile charged particles forming the current are actually embedded in a matrix, such as electrons in a metal wire. The ions that constitute the matrix tend to electrically neutralize the mobile particles and thus the conduction current is neutralized. Furthermore, the mobile source particles drift very slowly with respect to the matrix. Thereby, as shown in [7], the proposed electromagnetic force law (2) based on the augmented scalar potential can then be given in terms of the electric scalar potential Φ and the magnetic vector potential $\mathbf{A}$ by

$$\mathbf{F}(\mathbf{r}, t) = -q\nabla\Phi(\mathbf{r}, t) + q\nabla\left[\mathbf{v}_{em} \cdot \mathbf{A}(\mathbf{r}, t)\right] - q\left(\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r}, t)\right)_e, \tag{3}$$

where $\mathbf{v}_{em}$ is the velocity of the effector particle referred specifically to the matrix which in turn is supposed to move as a whole at a uniform velocity $\mathbf{v}_m$. The scalar and the vector potential in turn are given explicitly in terms of the net charge density ρ_n and the neutralized current density $\mathbf{J}_n$ respectively by the volume integrals

$$\Phi(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho_n(\mathbf{r}', t)}{R} dv' \tag{4}$$

and

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}_n(\mathbf{r}', t)}{R} dv', \tag{5}$$

where $\mathbf{J}_n = \rho_v\mathbf{v}_{sm}$, $\mathbf{v}_{sm}$ is the velocity of the mobile source particles referred specifically to the matrix, and the propagation time is neglected as the cases considered in this investigation are quasi-static.

By using Galilean transformations the preceding force law can be given by

$$\mathbf{F}(\mathbf{r}, t) = -q\nabla\Phi(\mathbf{r}, t) + q\nabla\left[\mathbf{v}_{em} \cdot \mathbf{A}(\mathbf{r}, t)\right] - q\left(\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r}, t)\right)_m - q\left(\mathbf{v}_{em} \cdot \nabla\right)\mathbf{A}(\mathbf{r}, t), \quad \tag{6}$$

where the time derivative $(\partial\mathbf{A}/\partial t)_m$ is referred to the matrix frame with respect to which the matrix is stationary. The terms $-q\nabla\Phi$ and $-q(\partial\mathbf{A}/\partial t)_m$ represent the electrostatic force and the electric induction force, respectively; meanwhile, the terms $q\nabla(\mathbf{v}_{em}\cdot\mathbf{A})$ and $-q(\mathbf{v}_{em}\cdot\nabla)\mathbf{A}$ represent the magnetostatic force and the magnetic induction force, respectively. By using a vector identity, the two force terms associated with the effector velocity $\mathbf{v}_{em}$ can be combined into the magnetic force. Thus the electromagnetic force exerted on an effector particle becomes a more familiar form

$$\mathbf{F}(\mathbf{r}, t) = q\left\{-\nabla\Phi(\mathbf{r}, t) - \left(\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r}, t)\right)_m + \mathbf{v}_{em} \times \nabla \times \mathbf{A}(\mathbf{r}, t)\right\}. \tag{7}$$

According to the dependences of the force terms on $\mathbf{v}_{em}$, one is led to express the force law in terms of the fields $\mathbf{E}$ and $\mathbf{B}$ in the form

$$\mathbf{F}(\mathbf{r}, t) = q\left\{\mathbf{E}(\mathbf{r}, t) + \mathbf{v}_{em} \times \mathbf{B}(\mathbf{r}, t)\right\}, \tag{8}$$

where the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ are then defined explicitly in terms of Φ and $\mathbf{A}$ as

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\Phi(\mathbf{r}, t) - \left(\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r}, t)\right)_m \tag{9}$$

and

$$\mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t). \tag{10}$$

The electromagnetic force law under the ordinary low-speed condition represents modifications of the Lorentz force law, which comply with Galilean transformations and can be in accord with the principle of relativity as the involved velocities are relative.

The fundamental modifications are that the current density generating the potential $\mathbf{A}$, the time derivative of $\mathbf{A}$ in the electric induction force, and the effector velocity connecting to $\nabla \times \mathbf{A}$ in the magnetic force are all referred specifically to the matrix frame. It is pointed out that this particular frame has been adopted tacitly in common practice dealing with the magnetic force, such as with the magnetic deflection. Further, the divergence and the curl relations for the corresponding electric and magnetic fields are derived. Apart from some minute deviation terms, these four relationships are just Maxwell's equations, with the exception that the velocity determining the involved current density and the associated time derivatives are also referred to the matrix frame [7].

3. Electromotance

We then go on to consider the electromotance (or called electromotive force) induced on a conducting wire in the presence of a solenoid. For a neutralized solenoid carrying a static current, both the electrostatic force and the electric induction force vanish. Thus the force contributes to the electromotance is the magnetic force

$$\mathbf{F} = q\mathbf{v}_{em} \times \mathbf{B}. \tag{11}$$

For the case where the solenoid is stationary and the wire is rotating at a rate ω with respect to the center z axis of the solenoid, the effector velocity associated with the magnetic force is $\mathbf{v}_{em} = \hat{z}\omega \times \mathbf{r}$, where $\mathbf{r}$ is the directed radial distance of the effector from the axis. Inside a long solenoid, the magnetic field $\mathbf{B}$ is known to be uniform. Thus the electromotance $\mathcal{V}$ induced on the wire C is given by

$$\mathcal{V} = \int_C \mathbf{v}_{em} \times \mathbf{B} \cdot d\mathbf{l} = \frac{1}{2}\omega B_0 r^2 \Big|_{r=r_a}^{r_b}, \tag{12}$$

where $\mathbf{B} = \hat{z}B_0$ with B_0 being a constant and r_a and r_b are the radial distances of the endpoints of the wire from the axis. Thus the electromotance is given by $\mathcal{V} = \omega(r_b^2 - r_a^2)B_0/2$.

For the case where the solenoid is rotating and the conducting wire is stationary, the velocity $\mathbf{v}_{em}$ varies among the various segments around the rotating solenoid, as the matrix velocity is no longer uniform. Consequently, the force law in terms of the vector potential $\mathbf{A}$ or the magnetic field $\mathbf{B}$ is not applicable. Instead, the general form of the magnetic force

$$\mathbf{F} = \frac{q\mu_0}{4\pi} \left\{ \nabla \left(\int \frac{\mathbf{v}_{em} \cdot \mathbf{J}_n}{R} dv' \right) - \int (\mathbf{v}_{em} \cdot \nabla) \frac{\mathbf{J}_n}{R} dv' \right\} \tag{13}$$

or

$$\mathbf{F} = \frac{q\mu_0}{4\pi} \int \mathbf{v}_{em} \times \nabla \times \frac{\mathbf{J}_n}{R} dv', \tag{14}$$

should be used, where the del operator applies only on the position vector $\mathbf{r}$ incorporated in R. For a thin wire C' carrying a conduction current I, the current density is in the direction tangent to the wire. Thus the electromotance is given by the double integral

$$\mathcal{V} = \frac{\mu_0 I}{4\pi} \int_C \left\{ \int_{C'} \nabla \left(\frac{1}{R} \right) \mathbf{v}_{em} \cdot d\mathbf{l}' - \int_{C'} (\mathbf{v}_{em} \cdot \nabla) \frac{1}{R} d\mathbf{l}' \right\} \cdot d\mathbf{l}, \tag{15}$$

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where the current is supposed to be static and uniform and is given by $I = \rho_l v_{sm}$, ρ_l is the line charge density of the mobile source particles (excluding the one of the neutralizing matrix), and $d\mathbf{l}'$ is parallel to $\mathbf{v}_{sm}$. The preceding formula may be difficult in calculation, as the matrix velocity and hence $\mathbf{v}_{em}$ are not fixed.

A simpler formula for electromotance can be obtained directly from the force law (3). Consider the electromotance induced on a conducting wire C due to a linear short element of a conduction current and of directed length $\mathbf{l}'$. Thus (3) leads to

$$\mathcal{V} = \frac{\mu_0 I}{4\pi} \left\{ \int_C (\mathbf{v}_{em} \cdot \mathbf{l}') \nabla \left(\frac{1}{R} \right) \cdot d\mathbf{l} - \int_C \frac{d}{dt} \left(\frac{\mathbf{l}'}{R} \right) \cdot d\mathbf{l} \right\}, \tag{16}$$

where R incorporated in the time derivative is varying with time whenever there is a relative motion between the wire and the current element and thus the time derivative is really associated with a quantity experienced by the effector on the wire. It is noted that for a linear short wire of directed length $\mathbf{l}$,

$$\frac{d}{dt} \left(\frac{\mathbf{l}'}{R} \right) \cdot \mathbf{l} = \frac{d}{dt} \left(\frac{\mathbf{l}' \cdot \mathbf{l}}{R} \right) - \frac{\mathbf{l}'}{R} \cdot \frac{d\mathbf{l}}{dt}. \tag{17}$$

The derivative $d\mathbf{l}/dt$ in turn corresponds to a rotation or deformation and is equal to the difference of velocity between the endpoints of the wire. The sum (designated as K) of the first integral in (16) and the last term in (17) can be given by

$$K = \tilde{\mathbf{v}} \cdot \mathbf{l}' \left(\frac{1}{R_b} - \frac{1}{R_a} \right) + \frac{1}{\tilde{R}} \mathbf{l}' \cdot (\mathbf{v}_b - \mathbf{v}_a), \tag{18}$$

where $\mathbf{v}_a$ and $\mathbf{v}_b$ denote the velocities of the endpoints a and b of the wire with respect to the current element, respectively, R_a and R_b are their distances from this element, and $\tilde{\mathbf{v}}$ or $\tilde{R}$ denotes a suitable mean value (say, the arithmetic average) between the endpoints. Then it is easy to show that when the directed length $\mathbf{l}$ approaches zero, one has

$$K = \frac{\mathbf{v}_b \cdot \mathbf{l}'}{R_b} - \frac{\mathbf{v}_a \cdot \mathbf{l}'}{R_a}. \tag{19}$$

Thus the electromotance due to the current element can be given by

$$\mathcal{V} = \frac{\mu_0 I}{4\pi} \left\{ \left. \frac{\mathbf{v}_{em} \cdot \mathbf{l}'}{R} \right|_a^b - \frac{d}{dt} \left(\frac{\mathbf{l}' \cdot \mathbf{l}}{R} \right) \right\}. \tag{20}$$

Thereby, the electromotance induced over a wire C due to a current-carrying wire C', both of arbitrary shape and length, is then given by

$$\mathcal{V} = \frac{\mu_0 I}{4\pi} \int_{C'} \left. \frac{\mathbf{v}_{em} \cdot d\mathbf{l}'}{R} \right|_a^b - \frac{d}{dt} \int_C \mathbf{A} \cdot d\mathbf{l}, \tag{21}$$

where a and b are the endpoints of C and the vector potential $\mathbf{A}$ is

$$\mathbf{A} = \mu_0 I \int_{C'} \frac{1}{4\pi R} d\mathbf{l}'. \tag{22}$$

It is noted that the formula for the vector potential with a nonuniform matrix velocity is still identical to the one with a uniform velocity, as the matrix velocity does not affect the

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drift speed. Thus the electromotance is composed of two terms, of which the one associated with $\mathbf{v}_{em}$ is primarily due to the magnetostatic force and the other is primarily due to the induction force.

Like the work done by a conservative force, the electromotance due to the magnetostatic force depends on the positions of the endpoints and not on the shape of the path connecting them. For a closed wire C, this term tends to vanish and thus the electromotance reduces to

$$\mathcal{V} = -\frac{d}{dt} \oint_C \mathbf{A} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s}, \qquad (23)$$

where S denotes the surface enclosed by the loop C, $d\mathbf{s}$ is in the direction normal to the surface, and the magnetic field is still related to the magnetic vector potential by (10), in spite of the matrix velocity being nonuniform. This formula looks like the one which is commonly cited to be related to Faraday's law of induction [11]. However, it is of essence to note that owing to the incorporation of the term $d\mathbf{l}/dt$ in (17), the time derivative in the preceding formula then operates on $d\mathbf{l}$ or $d\mathbf{s}$ and thus a rotation or deformation along with a translation of the loop contributes to the electromotance, as well as the time variation of $\mathbf{A}$ or $\mathbf{B}$ itself does.

Then consider the electromotance induced on a cylindrical capacitor in the presence of a solenoid C' of circular shape and carrying a uniform current I. Thus the resulting potential $\mathbf{A}$ is azimuthally symmetric. When either the solenoid or the capacitor is or both of them are rotating about the center axis of the solenoid, the integral of $\mathbf{A}$ in (21) is invariant with respect to time. Thereby, the electromotance becomes an even simpler form

$$\mathcal{V} = \frac{\mu_0 I}{4\pi} \int_{C'} \frac{1}{R} \mathbf{v}_{em} \cdot d\mathbf{l}' \Big|_a^b, \qquad (24)$$

where a and b denote two given points on the inner and outer cylinders, respectively, as depicted in Fig. 1. The actual path C implicitly associated with the preceding formula starts from point a and then traverses part of the inner cylinder, the connecting wires, and part of the outer cylinder, and finally comes to point b. However, since this electromotance does not differentiate the paths connecting the two endpoints, the electromotance over the actual path is then identical to the one over a complementary path connecting a to b directly via the space between the two cylinders. For the case where the solenoid is rotating and the capacitor is stationary, the relative motions between a given constituent segment of the actual path and different segments of the solenoid are different in direction. This situation is different from the case where the capacitor is rotating and the solenoid is stationary.

For each segment of a closely wound solenoid which is rotating, $\mathbf{v}_{em}$ is along the azimuthal direction and $d\mathbf{l}'$ is almost along it. Further, the directions of these two vectors change in a coordinated way around the solenoid and hence the product $\mathbf{v}_{em} \cdot d\mathbf{l}'$ in the preceding integral is a constant. Thus, for the stationary capacitor and connecting wires, the induced electromotance is given by

$$\mathcal{V} = -\frac{1}{2} \mu_0 r_0 \omega I N \left[L(r_b, z_b) - L(r_a, z_a) \right], \qquad (25)$$

where ω is the rotation rate, N is the number of turns of the solenoid, r_0 is its radius, r_a and z_a are the coordinates of the endpoint a in the cylindrical system, r_b and z_b are those of b, and L denotes the contour integral of $1/R$ over the spiral structure of the solenoid. Quantitatively, the dimensionless integral L can be given by

$$L(r, z) = \frac{r_0}{2\pi(z_2 - z_1)} \int_{z_1}^{z_2} \int_0^{2\pi} \frac{1}{\sqrt{(r_0 \cos \phi' - r)^2 + r_0^2 \sin^2 \phi' + (z' - z)^2}} d\phi' dz', \qquad (26)$$

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where the solenoid is supposed to be located between z_1 and z_2. It is seen that aside from a scaling factor the function L is identical to the electric scalar potential due to static charges distributed uniformly on the cylindrical surface fitting the solenoid.

The distributions of L as functions of the radial distance r with $z = 0$ and $z_1 = -z_2$ are shown in Fig. 2. When the solenoid is long, it is seen that the distribution of L is almost uniform with respect to r when $r < r_0$. Thereby, for a cylindrical capacitor placed inside a long rotating solenoid, the electromotance can vanish, as reported widely in the literature. On the other hand, it is of essence to note that outside the solenoid the distribution of L departures from being uniform. Consequently, it is predicted that a nonvanishing electromotance will be induced on the stationary capacitor when it is placed outside the rotating solenoid. Moreover, when the solenoid is not long, the distribution of L is nonuniform even when $r < r_0$. Thus a nonvanishing electromotance can also be induced on the capacitor when it is placed inside a short solenoid. In the experiment of Kennard with an outside capacitor, a nonvanishing electromotance was actually observed when the magnetization was reversed [3]. However, this experiment cannot discriminate between the electromotance due to the induction force with a time-varying current and the one due to the magnetostatic force, which are given by the second and the first term on the right-hand side of (21), respectively. However, a key difference between them is that the contribution due to the electric induction force occurs only at the instant of magnetization reversal and then disappears, while the one due to the magnetostatic force remains after the reversal. Thus it is possible to separate these two different kinds of electromotance with some auxiliary devices such as switches.

The predicted electromotance tends to accumulate positive and negative charges respectively on the two cylinders via the connecting wires. The induced charges in turn generate the electric scalar potential. At each point on the conducting plate, another scalar potential defined as the electromotance with respect to a certain reference point can be given unambiguously, as the electromotance due to the magnetostatic force and given by (24) does not differentiate the paths connecting the two points. Thereby, the distribution of charges can be determined by using the condition that the difference between such a magnetostatic potential and the electric scalar potential is fixed on the conducting plate. Thereby, an accurate evaluation of the charge density can be given by solving a relevant equation. Meanwhile, for a long solenoid the magnetostatic potential can be expected to be uniform longitudinally. From Fig. 3, it is seen that for a solenoid with a length of $10\,r_0$, the longitudinal distribution of the L function can be fairly uniform near the center of the solenoid with $|z| < r_0$. Then, by neglecting the fringing effect and the longitudinal variation of the magnetostatic potential, the total charge induced on either cylinder of a short capacitor is then given simply by $\pm Q$, where $Q = \mathcal{V}2\pi\epsilon_0 l/\ln{(r_b/r_a)}$, l is the length of the cylindrical capacitor, and $\mathcal{V}$ is the electromotance between two suitable points on the respective cylinders. The amount of the accumulated charges can be measured if suitable shielding and switches are provided, as in Barnett's experiment. This prediction of the nonvanishing electromotance and hence induced charges seems not to be reported before and then provides a means to test the validity of the modified force law.

For the case where both the solenoid and the capacitor are rotating, the situation is a little complicated, as the wires connecting the rotating capacitor to the electrometer are ordinarily stationary. Thus the path C implicitly associated with (24) should be divided into segments, over each of which its velocity is continuous as required in deriving (18). The electromotance over a stationary segment is the same as given before. Then we just consider a segment which as well as the solenoid is rotating at the same rate ω. Obviously, the electromotance as well as the velocity $\mathbf{v}_{em}$ of this case is the sum of those of the two cases where either the segment or the solenoid is rotating. Thus for a long solenoid the electromotance is expected to be still given by (12) when $r_0 > r_b > r_a$, and by (25) and (26) when $r_b > r_a > r_0$, where r_a and r_b are associated with the endpoints of the segment. Meanwhile, the situation can be quite different for the case with a short solenoid. Anyway, the electromotance can be given by numerical calculation. For the corotating case, the term

$\mathbf{v}_{em}\cdot d\mathbf{l}'$ is proportional to $\hat{r}'\cdot(\mathbf{r}' - \mathbf{r})$, where $\hat{r}'$ denotes a unit vector pointing from the center axis to the various segments of the rotating solenoid at a given instant. The electromotance is still given by (25) with the change that L corresponds to the integral of $\hat{r}'\cdot(\mathbf{r}' - \mathbf{r})/R$ and is then given by

$$L(r, z) = \frac{1}{2\pi(z_2 - z_1)} \int_{z_1}^{z_2} \int_0^{2\pi} \frac{r_0 - r\cos\phi'}{\sqrt{(r_0\cos\phi' - r)^2 + r_0^2\sin^2\phi' + (z' - z)^2}} d\phi'dz'. \qquad (27)$$

The distributions of L as functions of the radial distance r with $z = 0$ are shown in Fig. 4. It is seen that the numerical results agree with the preceding estimate of the electromotance.

4. Conclusion

Based on the modified Lorentz force law, which complies with Galilean transformations and depends on the relative velocity between the effector and the source particle, the electromotance over a conducting wire is derived. The electromotance is composed of two parts which are due primarily to the magnetostatic force and to the induction force, respectively. For a closed wire the electromotance becomes the time derivative of the linked magnetic flux, which is similar to the well-known formula related to Faraday's law except that the translation, rotation, and deformation of the loop have been taken into account explicitly. This time derivative can vanish due to some symmetry, such as in Barnett's experiment with a circular solenoid rotating about its center axis. Thus the electromotance reduces to the contribution due to the magnetostatic force and thus depends on the relative velocity between the effector and the matrix and on the positions of the endpoints of the wire. For the case with a long solenoid, the magnetostatic potential is uniform inside the solenoid. Thereby, the electromotance induced on a cylindrical capacitor placed inside a rotating solenoid vanishes. This agrees with Barnett's experiment. However, for the case with a short solenoid or for the case where the concentric capacitor is placed outside the solenoid, the electromotance can be different from zero. This prediction of nonvanishing electromotance then provides a means to test the validity of the modified Lorentz force law.

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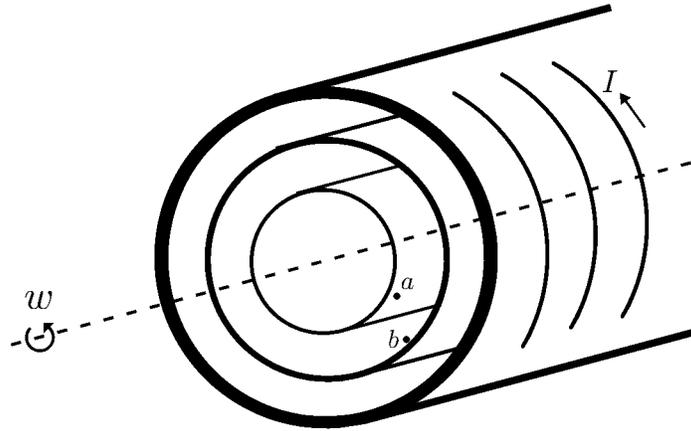
Fig. 1 Schematic of Barnett's experiment with a solenoid carrying a current I and a cylindrical capacitor. The inner and outer cylinders of the capacitor are coaxial with the solenoid and are connected to an electrometer with conducting wires and switches (not shown). The electromotance between point a on the inner cylinder and point b on the outer one is concerned.
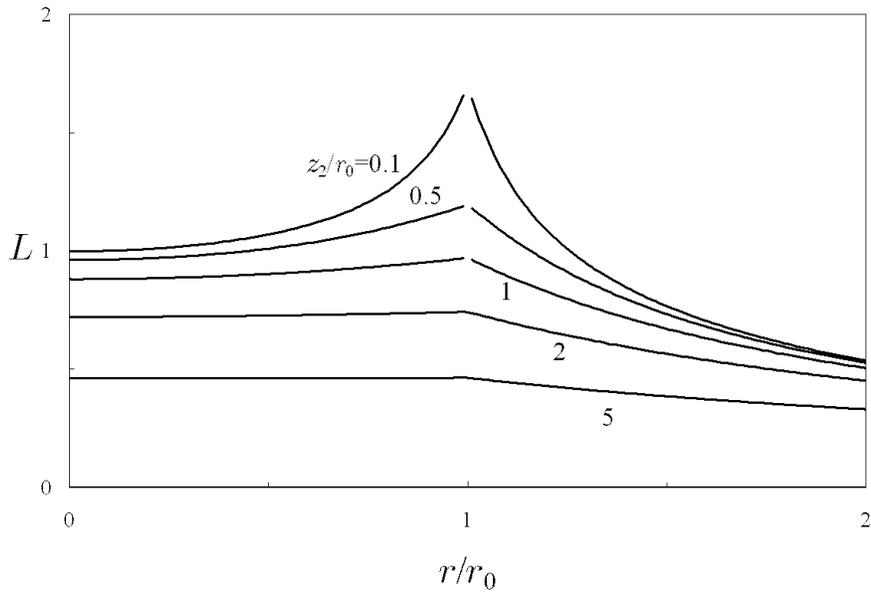
Fig. 2 Radial distribution of the L function at the center of the solenoid ($z = 0$) with its length ($2z_2$) as a parameter. The solenoid is rotating and the capacitor is stationary.
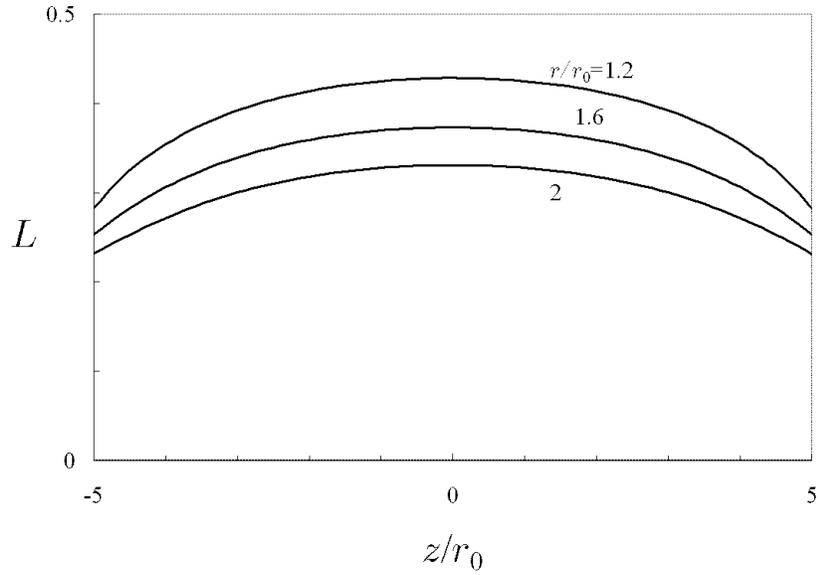
Fig. 3 Longitudinal distribution of the L function at $r = 1.2$, 1.6, 2 r_0. The solenoid is centered at $z = 0$ and of length 10 r_0.
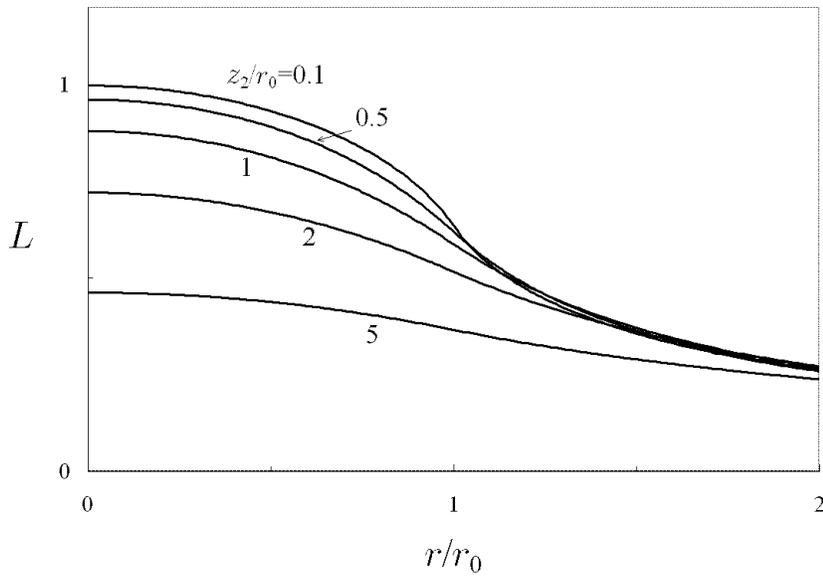
Fig. 4 Radial distribution of the L function at the center of the solenoid. Both the solenoid and the capacitor are rotating.

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