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Modifications of the Lorentz Force Law Invariant under Galilean Transformations

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Abstract — It is generally expected from intuition that the electromagnetic force exerted on a charged particle should be invariant as observed in different inertial frames. In the special relativity, this invariance is achieved by invoking the Lorentz transformation of space and time. In this investigation, an entirely different interpretation of this force invariance is presented by proposing a Galilean-invariant model of the electromagnetic force. In this new classical model, the electromagnetic force is expressed in terms of the augmented scalar potential. This new potential is a modification of the electric scalar potential by incorporating an ordinarily small velocity-dependent part. Each of the position vectors, time derivatives, and velocities involved in the proposed force law is referred specifically to a respective reference frame. By virtue of this feature, the electromagnetic force is endowed with the unique property of Galilean invariance. The velocity-dependent parts of the proposed force look quite different from their counterparts in the Lorentz force. However, under the common low-speed condition where the source particles forming the current drift very slowly in a matrix, the proposed Galilean-invariant model reduces to the Lorentz force law, if the latter is observed in the matrix frame as done in common practice.

I. Introduction

It is widely accepted that the electromagnetic force exerted on a particle of charge q and velocity $\mathbf{v}$ in the presence of electric and magnetic fields is given by the Lorentz force law as $\mathbf{F} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$. According to this famous law, the force consists of the electric and the magnetic forces. The electric force is associated with electric field and is independent of the motion of the charged particle; while the magnetic force is associated with magnetic field and depends on the particle velocity $\mathbf{v}$.

It is generally expected from intuition that a force exerted on a particle should be invariant as observed in different inertial frames. If the fields remain unchanged in different frames, then the invariance of electromagnetic force requires that the particle velocity $\mathbf{v}$ connecting to magnetic field should have a preferred frame. Historically, there were some disagreements about the reference frame of the particle velocity among Thomson, Heaviside, Lorentz, and Einstein who pioneered in the development of this important force law. However, in the past century, an almost unanimously accepted explanation is provided by Einstein in his original paper on the special relativity where the particle velocity $\mathbf{v}$ is referred to an arbitrary inertial frame. As observed from another reference frame, the relevant electromagnetic quantities of charge density, current density, potentials, and fields will change with the relative velocity between the two frames according to the Lorentz transformation of space and time. On the other hand, the kinematical quantities of velocity, momentum, and force will also change under the Lorentz transformation. It can be shown that the transformation of the time rate of change of kinematical momentum is exactly identical to the Lorentz force given in terms of the transformed fields and velocity [1]. In other words, the Lorentz force law is invariant under the Lorentz transformation.

In this investigation, an entirely different explanation of the invariance of electromagnetic force is presented. We will not invoke conceptually intricate space-time transformations as those adopted in the Lorentz transformation. Instead, a new classical model of electromagnetic force is proposed such that the electromagnetic force is invariant under Galilean transformations and reduces to a formula similar to the Lorentz force law under some ordinary situations. The proposed force law will be presented in terms of a new scalar potential, which is a modification of the electric scalar potential by incorporating a velocity-dependent part. Each of the position vectors, time derivatives, and velocities involved in the proposed model is referred specifically to some respective reference frame. By virtue of this simple feature, the electromagnetic force is endowed with Galilean invariance. How to relate the proposed Galilean model to the Lorentz force law is the key point and will be discussed in detail.

II. Augmented Scalar Potential and Proposed Force Law

In this section, we present a Galilean model of the electromagnetic force law given in terms of a single scalar potential $\breve{\Phi}$, which in turn is a modification of the electric scalar potential by incorporating a velocity-dependent part and is then called the augmented scalar potential. Consider an ensemble of mobile source particles of volume charge density ρ_v. The various source particles contribute collectively to potential which in turn will propagate to another charged particle under consideration called the *effector*. It is proposed that the *augmented scalar potential* $\breve{\Phi}$ experienced by an effector is given explicitly as

$$\breve{\Phi}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \left(1 + \frac{v_{es}^2}{2c^2}\right) \frac{\rho_v(\mathbf{r}', t - R/c)}{R} dv', \tag{1}$$

where $v_{es} = |\mathbf{v}_{es}|$, the velocity difference $\mathbf{v}_{es} = \mathbf{v}_e - \mathbf{v}_s$, $\mathbf{v}_e$ and $\mathbf{r}$ are respectively the velocity and position vector of the effector at the instant t, $\mathbf{v}_s$ and $\mathbf{r}'$ are those of the source at an earlier instant $t - R/c$, R/c is the propagation time from the source point $\mathbf{r}'$ to the field point $\mathbf{r}$, and the propagation range $R = |\mathbf{r} - \mathbf{r}'|$. It is supposed that speeds v_e, v_s, and hence v_{es} are much lower than c as they are ordinarily. Thus, the augmentation in the scalar potential is actually quite small. It is noted that the augmented scalar potential depends on the effector velocity and tends to be slightly different for different effectors.

Next, we address ourselves to the issue of the reference frames of the position vectors $\mathbf{r}$ and $\mathbf{r}'$ and of the propagation velocity of potential. Recently, we have proposed a Galilean model of wave propagation. It is supposed [2] that electromagnetic wave propagates via a medium like the ether. However, the ether is not universal. It is proposed that the region under sufficient influence of the gravity due to the Earth, the Sun, or another celestial body forms a local ether which in turn moves with the gravitational potential of the respective body. Thereby, each local ether together with the gravitational potential moves with the associated celestial body. Thus, as well as earth's gravitational potential, the earth local ether is stationary in an ECI (earth-centered inertial) frame. Consequently, the earthbound propagation depends on earth's rotation but is entirely independent of earth's orbital motion around the Sun or whatever. While, the sun local ether for the interplanetary propagation is stationary in a heliocentric inertial frame. This local-ether model has been adopted to account for a variety of propagation phenomena, particularly the GPS (global positioning system) Sagnac correction, the interplanetary radar echo time, and the earthbound and the interplanetary Doppler frequency shifts. Moreover, as examined with the present accuracy, the local-ether model is still in accord with the Michelson-Morley experiment which is known to make the classical ether notion obsolete. Further, by modifying the speed of light in a gravitational potential, this simple propagation model leads to the deflection of light by the Sun and the increment in the interplanetary radar echo time which are important phenomena predicted from the general theory of relativity [3].

Based on this new classical model, the position vectors $\mathbf{r}$ and $\mathbf{r}'$ and the propagation velocity of potential are referred specifically to the local-ether frame, which is an ECI frame for earthbound phenomena. In other words, as observed in the local-ether frame, the potential propagates isotropically at the speed c away from the emission position, independent of the motion of the source and the effector. As in the classical ether notion, the propagation time is the propagation range R divided by speed c and the propagation range in turn is the distance from the source at the instant of wave emission to the receiver at the instant of reception with the reference frame being attached to the local ether. Furthermore, the effector velocity $\mathbf{v}_e$ and the source velocity $\mathbf{v}_s$ are also referred specifically to the local-ether frame. Thereby, the proposed augmented scalar potential is Galilean invariant, since every position vector or velocity involved is referred to a specific reference frame.

Under the influence of the augmented scalar potential $\breve{\Phi}$ originated from an ensemble of mobile source particles, the electromagnetic force exerted on an effector of charge q and located at $\mathbf{r}$ at the instant t is hereby postulated to be

$$\mathbf{F}(\mathbf{r}, t) = q \left\{ -\nabla\breve{\Phi}(\mathbf{r}, t) + \left(\frac{\partial}{\partial t} \sum_i \hat{i} \frac{\partial}{\partial v_i} \breve{\Phi}(\mathbf{r}, t) \right)_e \right\}, \tag{2}$$

where $v_i = \hat{i} \cdot \mathbf{v}_e$, $\hat{i}$ is a unit vector, index $i = x, y, z$, and the time derivative $(\partial/\partial t)_e$ is referred specifically to the effector frame with respect to which the effector of velocity $\mathbf{v}_e$ is stationary. It is noted that the proposed force law resembles Lagrange's equations in classical mechanics. Physically, the time derivative $(\partial/\partial t)_e$ represents the time rate of change in some quantity experienced by the effector. As referred to a specific frame, the time derivative as well as the augmented potential is Galilean invariant. Consequently, the proposed electromagnetic force is endowed with the unique feature of Galilean invariance. In other words, this electromagnetic force is invariant when observed in different inertial frames, as expected intuitively. The proposed force law can be rewritten as

$$\mathbf{F}(\mathbf{r}, t) = q \left\{ -\nabla\breve{\Phi}(\mathbf{r}, t) - \left(\frac{\partial}{\partial t} \breve{\mathbf{A}}(\mathbf{r}, t) \right)_e \right\}, \tag{3}$$

where the *augmented vector potential* $\breve{\mathbf{A}}$ experienced by the effector is defined explicitly as

$$\breve{\mathbf{A}}(\mathbf{r},t)=\frac{1}{4\pi\epsilon_0 c^2}\int\frac{\mathbf{v}_{se}\rho_v(\mathbf{r}',t-R/c)}{R}dv'. \tag{4}$$

It is seen that potential $\breve{\mathbf{A}}$ is associated with minus the derivative of potential $\breve{\Phi}$ with respect to the effector speed v_e. These are our fundamental postulates for the Galilean-invariant electromagnetic force.

It is noted that both the augmented potentials depend on $\mathbf{v}_{es}$. Under quasi-static condition, the propagation delay R/c in the augmented potentials can be neglected and the velocity difference $\mathbf{v}_{es}$ approximates to Newtonian relative velocity between the effector and the source. Thereupon, for the pair of an effector and a single source, the proposed electromagnetic force is in accord with Newton's third law of motion: action is equal to reaction in magnitude.

Note that in the proposed force law the force term associated with the major part of $\nabla\breve{\Phi}$ is independent of particle velocities, while the other terms are velocity-dependent. The velocity-dependent force terms are expected to be much weaker than the velocity-independent term, since they incorporate the factor $1/c^2$. The velocity-independent term is identical to the conventional electrostatic force, as the propagation delay is neglected. However, the velocity-dependent force terms look quite different from the corresponding parts in the Lorentz force law. The similarity will emerge under the common low-speed condition where the source particles drift very slowly in a matrix, as discussed in the following sections.

III. Augmented Potentials in Matrix Frame

Consider the common case where the ensemble of mobile charged particles are flowing in a matrix. There are various types of the matrix, such as metal wires in antennas, semiconductors in light-emitting diodes, dielectric in lens, or magnet in motors. Ordinarily, the ions which constitute the matrix tend to electrically neutralize the mobile charged particles. Without the neutralizing matrix, the electrostatic force will be overwhelmingly dominant over the velocity-dependent force terms. Thus, the velocity-dependent forces are observable only under sufficient neutralization. Anyway, the matrix is supposed to have an arbitrary charge density ρ_m for generality. The neutralizing matrix can be not really in existence, for which case $\rho_m = 0$.

Ordinarily, the matrix is fixed in shape and moves as a whole at a uniform velocity $\mathbf{v}_m$ with respect to the local-ether frame. Thus, the various charges forming the matrix also move at this velocity. Then, due to the ensemble of mobile source particles and the matrix, the total electromagnetic force exerted on the effector is given by superposition. A little algebra leads to that the augmented potentials under neutralization take the forms of

$$\breve{\Phi}(\mathbf{r},t)=\Phi(\mathbf{r},t)\left(1+\frac{v_{em}^2}{2c^2}\right)-\mathbf{v}_{em}\cdot\mathbf{A}(\mathbf{r},t)+\frac{1}{2\epsilon_0 c^2}\int\frac{v_{sm}^2\rho_v(\mathbf{r}',t-R/c)}{4\pi R}dv' \tag{5}$$

and

$$\breve{\mathbf{A}}(\mathbf{r},t)=\mathbf{A}(\mathbf{r},t)-\frac{1}{c^2}\mathbf{v}_{em}\Phi(\mathbf{r},t), \tag{6}$$

where $\mathbf{v}_{em}=\mathbf{v}_e-\mathbf{v}_m$ and $\mathbf{v}_{sm}=\mathbf{v}_s-\mathbf{v}_m$. The electric scalar potential Φ and the magnetic vector potential $\mathbf{A}$ in turn are defined as

$$\Phi(\mathbf{r},t)=\frac{1}{\epsilon_0}\int\frac{\rho_n(\mathbf{r}',t-R/c)}{4\pi R}dv' \tag{7}$$

and

$$\mathbf{A}(\mathbf{r},t)=\frac{1}{\epsilon_0 c^2}\int\frac{\mathbf{J}_n(\mathbf{r}',t-R/c)}{4\pi R}dv', \tag{8}$$

where potential Φ is due to the net charge density $\rho_n=\rho_v+\rho_m$ and potential $\mathbf{A}$ is due to the *neutralized current density* $\mathbf{J}_n$ given as $\mathbf{J}_n(\mathbf{r},t)=\mathbf{v}_{sm}\rho_v(\mathbf{r},t)$. It is noted that the velocity of the charged particles forming the neutralized current density is referred to the matrix frame with respect to which the matrix is stationary and only the mobile source particles of charge density ρ_v contribute to this current. The velocity $\mathbf{v}_{sm}$ is Newtonian relative velocity between the mobile sources and the matrix and represents the drift velocity of mobile sources in the matrix. The neutralized current density is apparently Galilean invariant. Furthermore, potentials Φ and $\mathbf{A}$ defined in the preceding formulas are also Galilean invariant, as both $\mathbf{r}'$ and $\mathbf{r}$ are referred specifically to the local-ether frame. It is noted that unlike the augmented potentials $\breve{\Phi}$ and $\breve{\mathbf{A}}$, potentials Φ and $\mathbf{A}$ do not depend on the effector velocity and hence are identical for different effectors. Thus, it can be more convenient to express electromagnetic force in terms of Φ and $\mathbf{A}$.

IV. Force Law under Low-Speed Condition

Ordinarily, the mobile source particles drift very slowly with respect to the matrix. The drift speed of conduction electrons in a metal is normally lower than 1 mm/sec. Moreover, the speeds of terrestrial particles with respect to an ECI frame are normally of the order of the linear speed due to earth's rotation or less, which in turn is much lower than c. Thus, the various particle speeds are supposed to satisfy the *low-speed condition*:

$$v_{sm} \lll c \ \text{ and } \ v_e, v_{em} \ll c. \tag{9}$$

In words, this ordinary condition states that the effector, sources, and matrix move somewhat slowly with respect to the local-ether frame and the sources drift very slowly with respect to the matrix frame. Thereby, the augmented scalar potential becomes simpler as

$$\breve{\Phi}(\mathbf{r}, t) = \Phi(\mathbf{r}, t) - \mathbf{v}_{em} \cdot \mathbf{A}(\mathbf{r}, t). \tag{10}$$

Owing to the drift speed v_{sm} being very low, the matrix frame becomes the most convenient frame of reference. Moreover, it can be shown that the augmented vector potential can be approximated to the magnetic vector potential as $\breve{\mathbf{A}}(\mathbf{r}, t) = \mathbf{A}(\mathbf{r}, t)$.

Furthermore, based on Galilean transformations, the time derivative $(\partial \mathbf{A}/\partial t)_e$ referred to the effector frame can be given in the time derivative $(\partial \mathbf{A}/\partial t)_m$ referred to the matrix frame as $(\partial \mathbf{A}/\partial t)_e = (\partial \mathbf{A}/\partial t)_m + \mathbf{v}_{em} \cdot \nabla \mathbf{A}$, where the matrix frame is supposed to move at a fixed velocity $\mathbf{v}_m$ with respect to the local-ether frame. Then, by using a vector identity, the Galilean-invariant force law (3) can be given in terms of potentials Φ and $\mathbf{A}$ as

$$\mathbf{F}(\mathbf{r}, t) = q \left\{ -\nabla \Phi(\mathbf{r}, t) - \left(\frac{\partial}{\partial t} \mathbf{A}(\mathbf{r}, t) \right)_m + \mathbf{v}_{em} \times \nabla \times \mathbf{A}(\mathbf{r}, t) \right\}. \tag{11}$$

According to the dependence on effector velocity, one is led to express the force law in terms of fields $\mathbf{E}$ and $\mathbf{B}$ as

$$\mathbf{F}(\mathbf{r}, t) = q \left\{ \mathbf{E}(\mathbf{r}, t) + \mathbf{v}_{em} \times \mathbf{B}(\mathbf{r}, t) \right\}, \tag{12}$$

where electric and magnetic fields are defined explicitly in terms of potentials Φ and $\mathbf{A}$ as

$$\mathbf{E}(\mathbf{r}, t) = -\nabla \Phi(\mathbf{r}, t) - \left(\frac{\partial}{\partial t} \mathbf{A}(\mathbf{r}, t) \right)_m \tag{13}$$

and

$$\mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t). \tag{14}$$

This force law presents modifications of the Lorentz force law. The fundamental modifications are that the current density in potential $\mathbf{A}$, the time derivative applied to $\mathbf{A}$, and the effector velocity before $\nabla \times \mathbf{A}$ are all referred specifically to the matrix frame and that the propagation velocity of potentials is referred to the local-ether frame. Thereupon, the fields and the force are Galilean invariant. Furthermore, the Sagnac effect associated with the change in propagation delay due to the movement of effector with respect to the local-ether frame during wave propagation is proportional to v_e/c and hence is small [2]. Thereby, it is found from the derived force law in terms of fields that **under the ordinary low-speed condition, the proposed Galilean model is identical to the Lorentz force law, if the latter is observed in the matrix frame** as done in common practice.

V. Conclusion

A Galilean-invariant model of electromagnetic force is presented by proposing the augmented scalar potential, which is a slight modification of the electric scalar potential by incorporating a velocity-dependent part. Under the ordinary low-speed condition where the effector, sources, and matrix move slowly with respect to the local-ether frame and the sources drift very slowly with respect to a matrix frame, the proposed force law reduces to a form similar to the Lorentz force law. This presents modifications of the Lorentz force law. The fundamental modifications are that the current density in the vector potential, the time derivative applied to this potential in defining electric field, and the effector velocity connecting to the curl of this potential in the magnetic force are all referred specifically to the matrix frame and that the propagation velocity of potentials is referred to the local-ether frame. The proposed Galilean model is identical to the Lorentz force law, if the latter is observed in the matrix frame as done in common practice.

References

[1] P. Lorrain and D.R. Corson, *Electromagnetic Fields and Waves.* San Francisco: Freeman, 1972, chs. 5 and 6.

[2] C.C. Su, "A local-ether model of wave propagation complying with the Sagnac correction in the global positioning system," in *IEEE Antennas Propagat. Soc. Int. Symp. Dig.*, pp. 1570–1573, July 2000.

[3] C.C. Su, "A wave-propagation model for gravitational effects on light deflection and radar echo time," *J. Electromagnetic Waves Applicat.* (in press).